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THE ELECTRON MECHANICS OF INDUCTION ACCELERATION.

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ABSTRACT.

The radial and axial motions of electrons in the betatron are described by means of a potential function of forces. Previously reported conditions of equilibrium, stability and damping of oscillations are derived for the region of parabolic variation of the potential. Extension of the analysis to non-parabolic regions gives an account of the injection in conventional instruments in better agreement with experiment, particularly in regard to higher voltages of injection. Space charge limitations are discussed with the help of the Laplacian of the potential of forces. By means of an additional radial electric field electrons can be introduced as in the magnetron, without any asymmetry inherent in the conventional betatron circumferential injector. The analysis of the conditions of equilibrium and stability, greatly facilitated in this case by the notion of potential, shows that no substantial improvement in space charge limitations can be expected and that the required variations between the flux linking the electron orbits and the magnetic and electric fields at the orbits are difficult to realize on account of their complexity and narrow tolerances. The X-ray output of a small experimental double yoke instrument was measured by a phototube multiplier viewing an irradiated fluorescent screen and gave evidence of multiple group electron capture.

PART I.

INTRODUCTION.

The current widespread development of the betatron as a generator of high energy electrons and gamma rays has created need for a comparatively detailed knowledge of the electron dynamics of induction acceleration, both for the sake of engineering design and for possible future modifications and improvements. New points of view worked out for this purpose are applicable to other electronic devices such as the magnetron, or various electron optical lens systems of rotational symmetry. It is interesting to note that in the betatron are the longest, longest lived, and most stable, closed electron orbits known outside of the atoms themselves.

The idea of accelerating electrons by the solenoidal electric field of

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magnetic induction has been considered by many investigators (1, 2, 3, 6, 31).* In order for large energies to be gained in this manner, the acceleration must take place over very long electron paths, and while the magnetic field producing the electric field may simultaneously be used to bend the paths back on themselves, the elements of stability of these paths is of primary importance to successful operation. The basic principles concerning this stability were recognized by Steenbeck (17, 32), who published a design embodying most of the essential feature of present apparatus. These and other properties of the orbits, especially the mechanisms of electron injection, were discussed by Kerst and Serber (10), and the successful operation of a small betatron was reported by Kerst in 1940 (8).

The betatron or induction accelerator consists in its essentials of an alternating current magnet, generally with a laminated iron core [1] and an air gap, and within the gap an evacuated chamber containing an electron injector and a target. The injector, an electron gun, directs a beam of electrons circumferentially at a phase of the cycle of the magnet when the magnetic field is weak but increasing. The electrons, momentarily of moderate energies, are turned by the weak magnetic field into an approximately circular path while an electric field is induced along that path by the increasing flux of the magnet, a field which in ordinary transformers is responsible for the electromotive force induced in the windings. Therefore, as the electrons revolve in their orbits, they gain energy according to the number of turns they make. Meanwhile, the magnetic field increases too, so that in spite of the increased energy of the particles, they are held close to their original paths. Evidently, since the energy gain is determined by the average field intensity inside the circular path, while the field just at the circle keeps the electrons there, some definite relation between the two must exist. It turns out that a satisfactory relation is for the average field to be just twice the orbital field. Finally when the electrons have gained an energy corresponding to a change of flux of almost a quarter cycle, this two-to-one condition is artificially upset and the orbit moves toward the target. This is bombarded by the electrons and high energy gamma rays are produced. The gamma rays leave the chamber in the forward direction in a fairly well defined beam, but while some high speed electrons scattered by the target are capable of leaving the chamber, there is at present no published description[38] of a method for extracting the bulk of the electrons in a well defined beam.

It is intended here to develop the equations of electron motion as they are applied to induction acceleration, and with the aid of an arti-

* All such references will be found under heading "References" at end of Part II, May issue.

[1] Higher frequency air core systems may have desirable characteristics but have not yet been developed successfully (1, 22).

ficial potential function to encompass the earlier work on the electron trajectories and the conditions for stability, which constitute first order calculations for the most part, considerations on wider departures from equilibrium, particularly during injection, and the general influence of space charge. The application of an auxiliary electric field of rotational symmetry is discussed in detail without materially complicating the equations. Some experimental work complementing the theory is reported.

EQUATIONS OF MOTION.

The pertinent equation for electron motion in an electromagnetic field is given by

$$\frac{d}{dt}\{m\vec{v}\} = -e\{\vec{E} + \vec{v} \times \vec{B}\}, \tag{1}$$

wherein

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}} \quad \text{and} \quad \frac{e}{m_0} = 1.76 \times 10^{11} \text{ coulombs per kilogram.} \tag{2}$$

Here it is presumed at once that effects peculiar to quantum mechanical calculations are of no significance to the motion. Also, it is assumed that the acceleration of the electron is sufficiently small that associated force terms which appear in the general equation of motion are quite negligible. This assumption is substantially correct for all but the highest attainable accelerations for very high energy particles, a case which will be commented upon separately. The component equations of motion from (1), expressed in the conventional coordinates of the circular cylinder, are

$$\frac{d}{dt}[m\dot{r}] - mr\dot{\theta}^2 = -e[E_r + r\dot{\theta}B_z - \dot{z}B_\theta], \tag{3-r}$$

$$\frac{d}{dt}[mr^2\dot{\theta}] = -er[E_\theta + \dot{z}B_r - \dot{r}B_z], \tag{3-\theta}$$

$$\frac{d}{dt}[m\dot{z}] = -e[E_z + \dot{r}B_\theta - r\dot{\theta}B_r]. \tag{3-z}$$

The electric and magnetic field components expressed in these equations are of course related by the Maxwell electromagnetic field equations, and even though the sources of the field, i.e., charges and currents, may be of arbitrary distribution, two restrictive equations must be obeyed, namely:

$$\text{curl } \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0 \quad \text{and} \quad \text{div } \vec{B} = 0. \tag{4}$$

In the ideal betatron, the electric field is assumed to be generated solely by induction from the changing magnetic field, and it is without

sources such as charge or polarized materials. To the lines of the magnetic field are ascribed the symmetries of an axis of revolution and a plane of reflection. With the coordinate axis and the plane of the origin coinciding with these two respectively, it is also assumed that the angular component of the magnetic field, B_θ, and the angular variation of the axial component, $\partial B_z/\partial\theta$, are everywhere zero. From these hypotheses, it can be shown through equations (4) that of the field components only three remain, B_z which is a function of r, z, and t, and B_r and E_θ, which are related to B_z by the equations

$$rB_r = -\int_0^r r\frac{\partial B_z}{\partial z}\,dr \quad \text{and} \quad rE_\theta = -\int_0^r r\frac{\partial B_z}{\partial t}\,dr. \tag{5}$$

In vacuum, the remaining Maxwell field equation requires of B_z that

$$-\frac{1}{r}\int_0^r r\frac{\partial^2 B_z}{\partial z^2}\,dr - \frac{\partial B_z}{\partial r} + \frac{1}{c^2}\frac{1}{r}\int_0^r r\frac{\partial^2 B_z}{\partial t^2}\,dr = 0. \tag{6}$$

In the construction of an actual accelerator, the symmetry of the magnetic field is unavoidably imperfect and dielectrics and conductors associated with the vacuum chamber disturb the induced electric field. The error made in the forthcoming analysis by neglecting the small forces arising from the angular dissymmetries which may be present, depends upon whether the principal forces are in unstable equilibrium and upon whether the small effects are likely to be cumulative. Some tentative judgment on these questions will become possible as a result of the analysis of the ideal case.

With the assumption of complete symmetry in the fields as stated, equations (3) reduce, with the aid of equations (5), to a comparatively simpler set.

$$\frac{d}{dt}[m\dot{r}] = mr\dot{\theta}^2 - er\dot{\theta}B_z, \tag{7-r}$$

$$\frac{d}{dt}[m\dot{z}] = -e\dot{\theta}\int_0^r r\frac{\partial B_z}{\partial z}\,dr, \tag{7-z}$$

$$\frac{d}{dt}[mr^2\dot{\theta}] = e\left[\int_0^r r\frac{\partial B_z}{\partial t}\,dr + \dot{z}\int_0^r r\frac{\partial B_z}{\partial z}\,dr + \dot{r}B_z\right]$$

$$= e\frac{d}{dt}\int_0^r r\,B_z dr. \tag{7-\theta}$$

Recalling the functional dependence of B_z, it is noted that

$$2\pi\int_0^r rB_z(r, z, t)dr$$

is the magnetic flux passing through the circle r, z. The integration of

equation (7-θ) calls for a constant, C, which describes some of the initial circumstances upon injection of the electron into the field.

$$C = \frac{m^0}{e} r_0^2 \dot{\theta}_0 - \int_0^r r B_z(r, z_0, t_0) dr, \tag{8}$$

wherein the superscript and subscripts 0 denote the initial values of the quantities. Thus the integral of equation (7-θ) is

$$r\dot{\theta} = \frac{e}{m} \frac{\int_0^r r B_z dr + C}{r}. \tag{9}$$

If this result is placed in equations (7-r) and (7-z), it is found that

$$\frac{d}{dt}[m\dot{r}] = -\frac{e^2}{2m}\frac{\partial}{\partial r}\left[\frac{\int_0^r r B_z dr + C}{r}\right]^2, \tag{10-r}$$

$$\frac{d}{dt}[m\dot{z}] = -\frac{e^2}{2m}\frac{\partial}{\partial z}\left[\frac{\int_0^r r B_z dr + C}{r}\right]^2. \tag{10-z}$$

Were it not for the variance of the mass, m, the form of equations (10) would be strongly suggestive of two-dimensional potential motion. However, the mass, or the total energy of the particle, can change only through motion along the one existing component of the electric field, E_θ. Thus from equation (5) and (9), the mass-energy equivalence gives for the total variation of mass:

$$\frac{d}{dt}[mc^2] = -er\dot{\theta}E_\theta = -\frac{e^2}{2m}\frac{\partial}{\partial t}\left[\frac{\int_0^r r B_z dr + C}{r}\right]^2 \tag{11}$$

and the value of m is given directly by combining equation (9) with the formula for the mass, (2),

$$m^2c^2 = \frac{e^2\left[\frac{\int_0^r r B_z dr + C}{r}\right]^2 + m_0^2c^2}{1 - \frac{\dot{r}^2 + \dot{z}^2}{c^2}}. \tag{12}$$

The particle's kinetic energy is thus:

$$V = \frac{(m - m_0)c^2}{e} = \frac{m_0c^2}{e}\left\{\sqrt{\frac{\frac{e}{m_0}\left[\frac{\int_0^r r B_z dr + C}{r}\right]^2}{\frac{m_0c^2}{e}\left[1 - \frac{\dot{r}^2 + \dot{z}^2}{c^2}\right]} + 1} - 1\right\}. \tag{13}$$

In virtually all operable betatrons, the number of "volts per turn" is very small in comparison to the total energy of the electron, starting at $V_0 = .512$ Mev rest energy, so that in the course of a few revolutions the proportional change in m as it appears in equations (10) is very small. The dependence of this change on the radial and axial trajectory as in equation (11) is an order of magnitude smaller still and may be neglected altogether.

The differentiation on the left of equations (10) leads to the terms $\dot{r}(dm/dt)$ and $\dot{z}(dm/dt)$ having the character of viscous damping terms, because the increasing mass of the accelerated electron tends to bring about a reduction in the radial and axial velocities of its motion. Again for conventional instruments having a small number of "volts per turn," and this includes practically all low frequency magnets, these terms are very small and for most purposes may be neglected. During the early, low energy stages of the acceleration, where damping effects are of particular interest, there are other very much larger ones to be discussed, which arise through the changing of the magnetic field. Although the initially larger terms tend to vanish at relativistic speeds and leave the mass variation terms as the only damping, the latter will be ignored in most of this discussion.

Under the above provisions, equations (10) can be written exactly as if describing two-dimensional potential motion.

$$m_0\ddot{r} = -e\frac{\partial}{\partial r}\left\{\frac{em_0}{2m^2}\left[\frac{\int_0^r rB_z dr + C}{r}\right]^2\right\} = -e\frac{\partial}{\partial r}V_M, \qquad (14\text{-}r)$$

$$m_0\ddot{z} = -e\frac{\partial}{\partial z}\left\{\frac{em_0}{2m^2}\left[\frac{\int_0^r rB_z dr + C}{r}\right]^2\right\} = -e\frac{\partial}{\partial z}V_M, \qquad (14\text{-}z)$$

wherein the symbol V_M, the "potential of magnetic and inertial forces," is introduced for brevity. Also for convenience let the symbol V_{M0} replace $(m^2/m_0^2)V_M$. With this notation, an approximation to equation (13), useful for electrons whose tangential component of velocity is the principal one, is

$$V \cong V_0\left[\sqrt{\frac{2V_{M0}}{V_0} + 1} - 1\right], \qquad (15)$$

the limiting forms of which are, for low energies, V_{M0}, and for very high energies, $\sqrt{2V_0V_{M0}}$.

If, in addition to the magnetic and induction fields, an electrostatic potential field V_E having the same symmetries as the magnetic field is applied to the region of the electron motion, and thus superposed on the fields which have already been considered, the angular motion equa-

tions (7-θ) and (9) remain unaltered, while to the right sides of equations (10) for the radial and axial motions are added the appropriate forces obtained from the gradient of V_E, namely, $e(\partial V_E/\partial r)$ and $e(\partial V_E/\partial z)$. In this case, however, the change in mass does not occur only through motion in the tangential or angular direction, so that in order to achieve the same simplicity as in equations (14), the new equations must be restricted to the non-relativistic region. For most purposes, this is not a severe restriction, since electrostatic potentials large enough to materially influence the motion of the higher speed electrons are hard to apply. In conventional accelerators, the additional electric field is due primarily to the space charge of the electrons in the chamber and is of importance only in the early stages of acceleration. The equations of motion corresponding to equations (14) now have the form:

$$m_0\ddot{r} = -e\frac{\partial}{\partial r}[V_M - V_E],\ m_0\ddot{z} = -e\frac{\partial}{\partial z}[V_M - V_E]. \qquad (16)$$

THE TWO DIMENSIONAL MOTION

The approximate equivalence of the radial and axial electron motion to two-dimensional classical motion in a potential field slowly changing with time offers important advantages, both for the analytical and the intuitive understanding of the processes involved. As has already been stated, the total electron path is very long, and while it is turned back upon itself, the region in r and z which it may occupy, must of course be within the vacuum chamber. To thus confine the electron, there must be, in this region, a minimum in the potential surface V_M, or $V_M - V_E$ if the additional electric field is present, and the two-dimensional analog of the particle's kinetic energy must be insufficient to carry it beyond the potential barrier surrounding this minimum which corresponds to a stable circular orbit. With the particle kept in this bowl, the necessary properties of the trajectory are obtained. For purposes of discussion, these properties may be subdivided according to whether they depend on the momentary existence and shape of the bowl or whether they pertain to the effects of the change of this shape with time.

The elementary conditions for the existence of an instantaneous stable circular orbit, an "instantaneous equilibrium orbit" of radius r_s, may be found directly by applying the conventional relations of analytic geometry to the function $[V_M - V_E]$.

(a) $$\frac{\partial}{\partial r}[V_M - V_E] = 0$$

(b) $$\frac{\partial^2}{\partial r^2}[V_M - V_E] > 0$$

(c) $$\frac{\partial^2}{\partial z^2}[V_M - V_E] > 0$$

(d) $$\frac{\partial}{\partial z}[V_M - V_E] = 0$$

(e) $$\frac{\partial^2[V_M - V_E]}{\partial r^2}\frac{\partial^2[V_M - V_E]}{\partial z^2} - \left\{\frac{\partial^2[V_M - V_E]}{\partial r \partial z}\right\}^2 > 0. \qquad (17)$$

On account of the symmetry which the electric and magnetic fields possess with respect to the plane of $z = 0$, the fourth condition is satisfied everywhere on this "equilibrium orbit plane." It then follows that on this plane the last condition is satisfied if the second and third are, so that only the discussions of the first three conditions, called the radial equilibrium, the radial and the axial focusing, conditions, respectively, are not trivial. If V_E is taken to be absent, the radial equilibrium condition applied to V_M gives the alternative relations

$$\frac{\int_0^r rB_z dr + C}{r} = 0, \qquad (18)$$

$$B_z - \frac{\int_0^r rB_z dr + C}{r^2} = 0. \qquad (19)$$

The first of these refers to a condition for the electron at rest, see equation (9), and is of no interest here. The second alternative is the appropriate condition for equilibrium in the radial direction. If it is included in the formulas when the radial and the axial focusing conditions are applied to V_M, the following two inequalities are obtained, respectively (32):

$$\frac{r}{B_z}\frac{\partial B_z}{\partial r} > -1, \qquad (20)$$

$$B_z \int_0^r r \frac{\partial^2 B_z}{\partial z^2} dr > 0, \qquad (21)$$

wherein $\partial B_z/\partial z$ is zero for $z = 0$. Taking into account the quasi-static nature of the magnetic fields with which one must deal in practice, the time derivative term in equation (6) may be neglected and the result used for substitution in relation (21) which may be written with the same functional expression as (20). Combining the two:

$$1 > \left[-\frac{r}{B_z}\frac{\partial B_z}{\partial r} \right] > 0. \qquad (22)$$

This means simply that the lines of the magnetic field must be convex

outward and that the field intensity diminishes with increasing radius, while the corresponding diminishing in centripetal magnetic force must occur more slowly than the decrease in centrifugal force at constant electron velocity. The quantity in brackets is a more general form of the "n" of Kerst and Serber who assumed that the magnetic field was proportional to r^{-n}. During the acceleration cycle the importance of the constant C becomes less and less relative to the time increasing $\int_0^r rB_z dr$. Equation (19) approaches the condition,

$$B_z = \frac{\int_0^r rB_z dr}{r^2}, \tag{23}$$

wherein the right side is obviously $\frac{1}{2}$ the average value of B_z within the circle. To have a stable orbit, condition (23), together with condition (22), must be satisfied for some value of r. In conventional instruments, the magnetic field intensities at all points vary synchronously throughout the greater portion of the acceleration cycle, and so this value of r, called the principal equilibrium orbit, is a constant, r_e. Then, as may be noted by applying the present conditions to the formula from (5), this position is also the minimum for E_θ or the equivalent, volts per turn divided by radius. Kerst and others have used this fact for measuring the position of the equilibrium orbit.

Induction accelerators are usually built so that condition (21) is satisfied for all positions within the vacuum chamber, and since the equilibrium orbit plane $z = 0$ is one of symmetry, the axial sections of the potential surfaces V_M have substantially similar characteristics whatever the other circumstances may be. They are parabola-like in shape, symmetrical and increasing outward monotonically on either side of $z = 0$. The rate of the increase depends on how negative is $\frac{r}{B_z}\frac{\partial B_z}{\partial r}$, and the adjustment of this parameter determines whether the walls of the potential bowl, in the axial direction, are sufficient to keep the electrons from the corresponding material walls or other obstructions of the vacuum chamber. While the radial sections of the potential surfaces are of no greater intrinsic importance than the axial ones, the appearance of the constant C in the formula for V_M makes their characteristics critically dependent on the initial conditions. For each value of C, the radial sections are, for the most part, of the same general shape for different values of z. A representative set of such radial sections of V_M for different values of C for the plane $z = 0$, and which thus includes the equilibrium orbit position, is given in Fig. 2. The instantaneous magnetic field distribution from which these curves were computed is shown in Fig. 1, which includes an outline of the pole pieces generating this field.

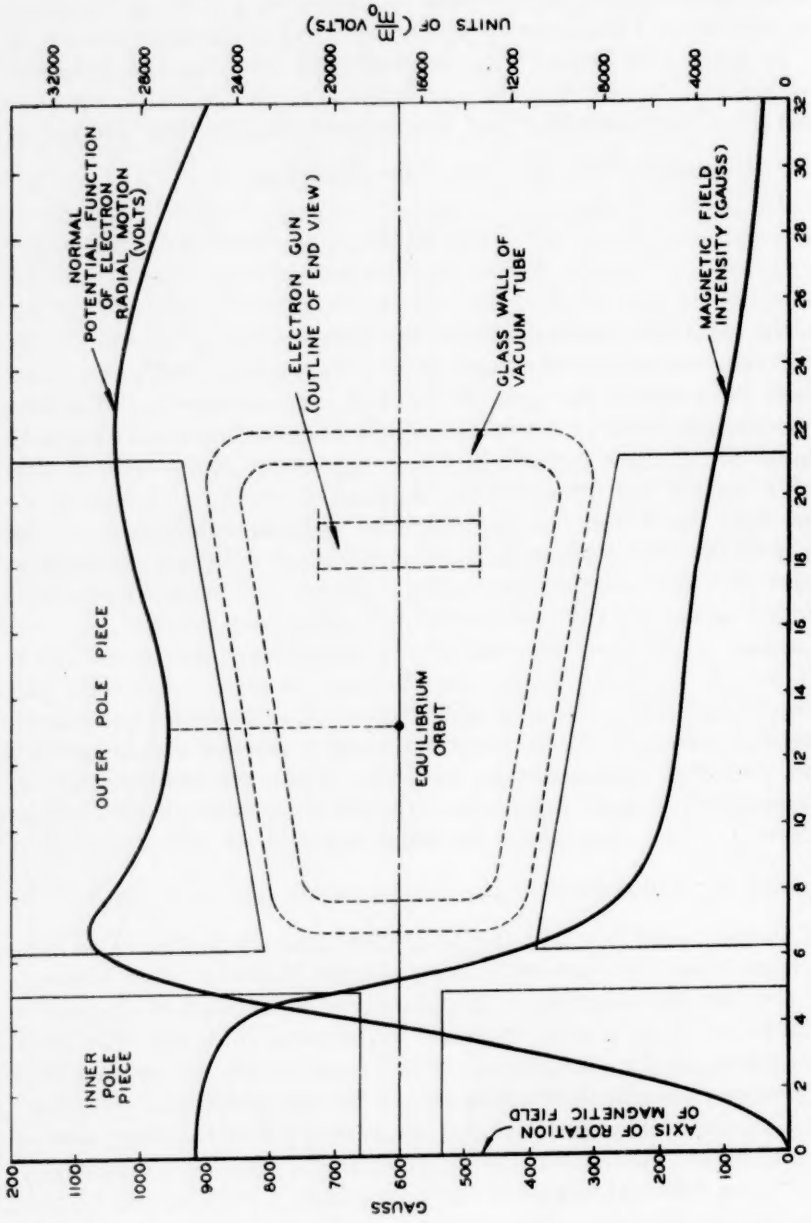

FIGURE 1. MAGNETIC FIELD CURVES.

FIG. 1. Radial variation of magnetic field.

A large variety of injection phenomena can be understood by consideration of the V_M radial section curves only, since usually the axial sections may be taken for granted. While the curves of Fig. 2 are entirely representative of all values of C, the particular ones drawn were chosen with the position $r = 18$ in mind as the initial location of the electrons. In such an instance, these particular values of C correspond to tangential velocities of injection (neglecting relativity corrections) differing by a simple factor, one plus the coefficient of P_{18}, from the velocity of the "normal" electrons, whose C value is zero and hence whose equilibrium orbit radius is unchanged during the acceleration. With reference to the concept of two-dimensional conservative motion, it is a matter of inspection of Fig. 2, to see that for electrons injected at $r = 18$, even for those without radial and axial velocities, only those with tangential velocities corresponding to C values within the interval $-.020\,P_{18}$ to $+.125\,P_{18}$, approximately, can be captured initially within the potential bowls. For those with small radial velocities in addition, the range is evidently more restricted and can be found immediately by recognizing the additional barrier necessary to contain the additional "two-dimensional kinetic energy." While the potential minimum for which V_M is zero has been excluded from consideration on the conventional betatron because the associated acceleration is small, it represents the situation of zero tangential velocity obtained when electrons are emitted directly into the chamber from a hot cathode located at the zero position, such as $r = 18$ for $C = 1.0\,P_{18}$. With the addition of the electrostatic field V_E from a concentric anode, this case describes a static magnetron with a non-uniform magnetic field and is considered in detail below.

FIGURE 1. MAGNETIC FIELD CURVES.

FIG. 1. Radial variation of magnetic field.

If an electron in a conventional betatron is confined near to a stable circular orbit in accordance with the existence of a minimum in the appropriate surface V_M, there are associated with its motion three frequencies or periods which are more or less unique, depending on whether the radial and axial oscillations are of small or large amplitude. In the limit of very small such amplitudes the periods of rotation and radial, and axial oscillation are, respectively:

$$T_\theta = 2\pi \left[\frac{e}{m} B_z \right]^{-1}, \qquad \text{at } r = r_i \tag{24}$$

$$T_r = 2\pi \left[\frac{e}{m} B_z \right]^{-1} \left[1 + \frac{r}{B_z} \frac{\partial B_z}{\partial r} \right]^{-\frac{1}{2}}, \qquad \text{at } r = r_i \tag{25}$$

$$T_z = 2\pi \left[\frac{e}{m} B_z \right]^{-1} \left[-\frac{r}{B_z} \frac{\partial B_z}{\partial r} \right]^{-\frac{1}{2}}. \qquad \text{at } r = r_i \tag{26}$$

Provided the particle remains within the potential bowl for large amplitudes, only the formula for T_r needs major correction: to be multiplied

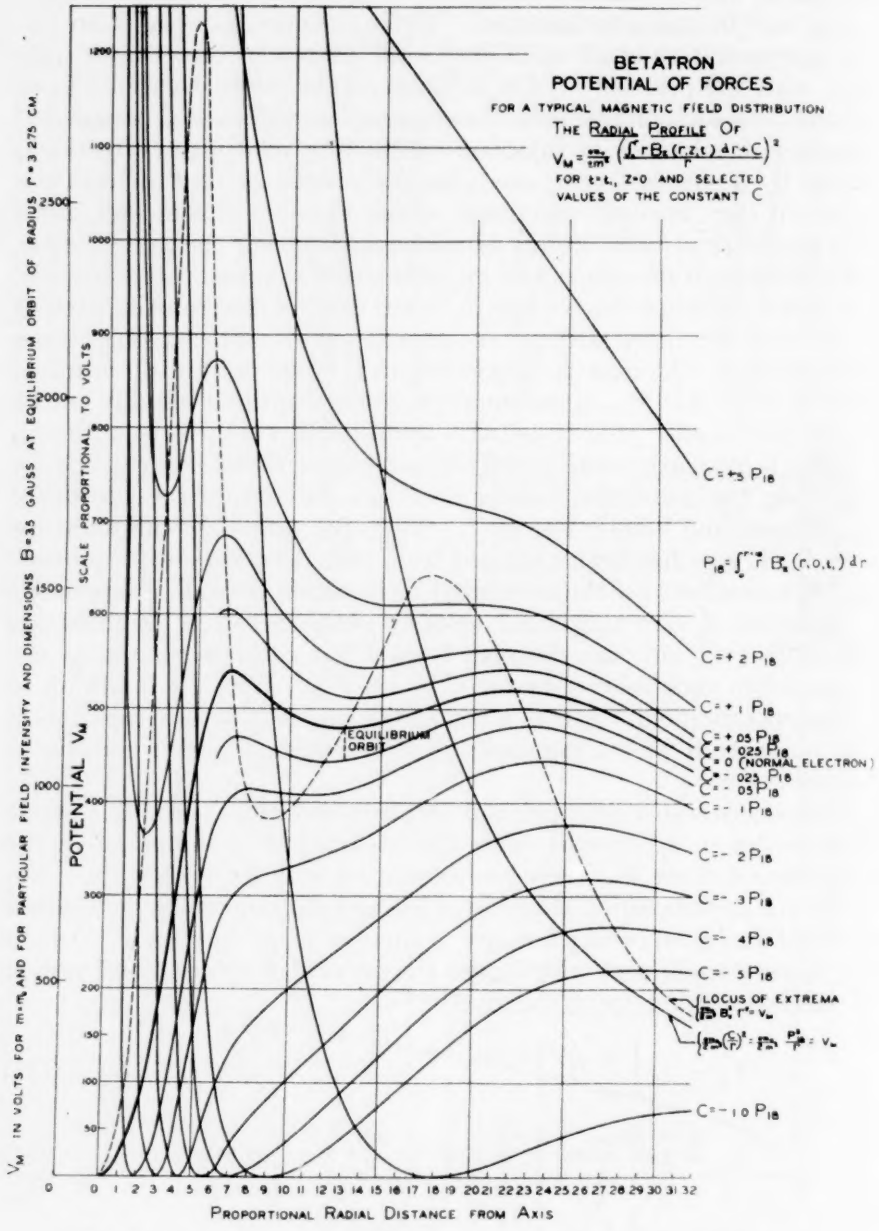

FIG. 2. The radial profiles of the potential of magnetic and inertial forces.

by a factor much greater than unity because the particle approaches the rounded crests of the surrounding potential barrier. In any event, since $\frac{r}{B_z}\frac{\partial B_z}{\partial r}\Big|_{r=r_i}$ obeys condition (22), the periods T_r and T_z are always greater than T_θ.

Very small angular dissymmetries of magnetic and induced electric field strengths may be present in an accelerator. If it is presumed that the electron motions are very nearly those of the ideally symmetrical case, the potential notion may still be applied, and the small dissymmetries can be accounted for by small, additional radial and axial forces, which recur with a period of T_θ. Since T_r and T_z are greater than T_θ, it follows that no direct resonance effect is likely to occur and that the disturbances on the ideal motion are not likely to accumulate beyond what are produced in the first few revolutions after injection. Thus, if the electron is well contained in the potential bowl in the first place, the motion will be in accordance with the formulas developed for symmetrical fields.

SPACE CHARGE.

Whether an electrostatic field is applied from electrodes or not, a large space charge will contribute to V_E and should be taken into account. Since the charge distribution is not known and in general will depend on the manner of injection, the stability conditions (17), which still must be satisfied, are not capable of direct verification. They lead, however, to a definite limitation on the supportable space charge density, for by summing the first three of conditions (17), [(17a) being multiplied by $1/r$], one finds that the Laplacian of $[V_M - V_E]$ must exceed zero. Since V_E itself must obey Poisson's equation, it follows that:

$$-\frac{\rho}{\epsilon_0} < \nabla^2 V_M = \frac{em_0}{m^2}\left\{\left[B_z - \frac{\int_0^r rB_z dr + C}{r^2}\right]^2 + \left[\frac{\int_0^r rB_z dr + C}{r^2}\right]^2 + \left[\frac{\int_0^r r\frac{\partial B_z}{\partial z}dr}{r}\right]^2\right\}. \quad (27)$$

The limiting charge density given by this relation may be found in charge distributions in complete axial and radial equilibrium, wherein the repulsive forces between particles are just compensated by the retaining forces of the applied fields. Consideration of an optimum of such distributions will establish the upper limit of charge supportable in the betatron, whatever the injection mechanism. The magnetic forces of the space charge current are assumed negligible at the time of injection.

Supposing at first that all electrons present have a common value for the integration constant C, the surface of $[V_M - V_E]$ which originally possessed a minimum before the electrons were inserted and space charge forces contributed to V_E, now has the shape of a bowl whose bottom is flat over the region of the charge distribution. Here the charge density is given by the right side of relation (27) and as more charge is added, this value is not exceeded but the distribution extends itself over a larger space. Eventually, with additional charge, the repulsive forces can no longer be counterbalanced. This is shown in the surrounding potential barrier, where, at least at one point on the edge of the flat bottom, the barrier is pressed down, as the surface curvature changes from concave upward to concave downward. Then no further charge can be supported for it spills out by this route. With the usual field shapes, this "leak" will occur in a radial direction in the $z = 0$ plane provided the vacuum chamber walls and other obstructions have not interfered beforehand, because the potential surface goes to relatively greater height in the axial direction. It seems fairly certain that in this case of equilibrium, there is supported the greatest possible number of electrons with the chosen value of C. The configuration is one of minimum total energy as compared to non-equilibrium distributions of the same amount of charge, and these distributions in sufficient time must surely rearrange themselves so that some of the particles will surmount the surrounding barrier by virtue of their extra energy, even if, as is unlikely, the initial arrangements succeed in reducing the space charge repulsions relative to the retaining forces.

If groups of electrons of different values of the integration constant C are present and in complete equilibrium (without orbital oscillations), they cannot mingle in the same spatial region because these values appear explicitly in the radial equilibrium condition (17a) which cannot be satisfied for the same space coordinates but different C's. It follows from the continuous dependence of the equilibrium relations on C that although one group may be space charge limited with its potential bowl depressed at some point, in general another group may be found of slightly higher or lower C whose potential surface $[V_M - V_E]$ still has a minimum and can receive more charge. Assuming the injector is able to provide it, the additional space charge will cause some of the previous group to be lost, but the equilibrium position of the new group, as given by (17a) with the new value of C, is necessarily well removed from the position of the "leak" in the old group. Then on members of the old group at this position, the proportional repulsion, or force per unit charge, of the new group is less by virtue of greater distance than was that of the displaced charge, and there is a net gain in the total charge supported. If the injection mechanism is able to meet the demand, this displacement of groups may go on over a continuous range of C values until that group is reached which simultaneously de-

presses its potential barrier in two places of different radii, both radially outward and inward for conventional instruments. Now no further group of slightly different C can have a minimum in its potential surface and the space charge limited condition of the last named group, in equilibrium and standing alone, is the final configuration.

For the above reasons, it is thought that the optimum space charge saturated condition of the betatron, which can be obtained at the time of injection, consists of the support in radial and axial equilibrium of the full complement of one group of electrons of a single value of C, or in most cases of a single tangential velocity and time of injection. Since the influence of the space charge field depends not only on the charge distribution but also on the electrostatic boundary conditions which must be satisfied, the exact computation of the value of the integration constant C which applies to the select electron group is difficult, if not impossible. Fortunately, there is a wide latitude in the choice of this C which will lead to substantially the same total for the supported space charge, so that one may assume a value corresponding in the case of a conventional betatron to the radial profile curve of V_M whose two maxima are of the same height, on the grounds that there will be a rough symmetry about the minimum position. In Fig. 2, this value is about $C = -.008\ P_{18}$. It should be emphasized, however, that the injection mechanism may be quite unable to establish this optimum electron group and it may be necessary to consider another group of very different value of C, which can support much less charge.

To find the upper limit of the capturable charge itself, the space charge density given by equation (27) with a proper choice of C, must be integrated within the limits of the charge distribution. In estimating these limits, electrostatic boundary conditions must be taken into account. If the induced space charge image fields are assumed to be negligible, the radial curvature of the negative space charge field alone will be negative throughout the distribution. Thus, by radial focusing condition (17b), the charge distribution cannot extend beyond the points of inflection of the radial profile curve of V_M and probably not quite as far. A fair guess of the limits can be made by taking the lower inflection point for one limit and its horizontal projection to the other side of the bowl of V_M for the other. In Fig. 2, these limits are, for $C = .008\ P_{18}$, $r_2 = 16$ and $r_1 = 9$, and for $C = +.1P_{18}$, $r_2 = 16.5$ and $r_1 = 15.5$. The relative axial to radial extents of the charge distribution is determined to a first approximation by the square root of the inverse ratio of curvatures of V_M. Evaluated at the instantaneous equilibrium orbit $r = r_i$, these values are

$$\frac{\partial^2 V_M}{\partial z^2} = -\frac{em_0}{m^2} B_z^2 \frac{r}{B_z} \frac{\partial B_z}{\partial r}, \qquad \text{at } r = r_i \qquad (28\text{-}z)$$

$$\frac{\partial^2 V_M}{\partial r^2} = \frac{em_0}{m^2} B_z^2 \left[1 + \frac{r}{B_z} \frac{\partial B_z}{\partial r} \right] \cdot \quad \text{at } r = r_i \tag{28-r}$$

The experimental adjustment of the parameter $\frac{r}{B_z}\frac{\partial B_z}{\partial r}$ to from $-\frac{3}{4}$ to $-\frac{1}{2}$ in accordance with condition (22) so as to have the electron configuration roughly conform with the shape of the vacuum chamber leads in present instruments to an axial extent from $\frac{1}{2}$ to 1 times the radial extent for values of r_i close to the main equilibrium orbit, but very much smaller ratios for other electron groups. The radial-axial distribution having an approximately elliptical boundary, one finds the total charge, Q, supported in this toroidal ring is the product of the volume and the average charge density of (27) which is roughly given by its value at r_i.

$$Q = \pi^2 \epsilon_0 r \left\{ \frac{[r_2 - r_1]^2 \left[1 + \frac{r}{B_z}\frac{\partial B_z}{\partial r} \right]^{\frac{1}{2}}}{r^2 \left[-\frac{r}{B_z}\frac{\partial B_z}{\partial r} \right]^{\frac{1}{2}}} \right\} \left\{ \frac{em_0}{2m^2} \left[\frac{\int_0^r r B_z dr + C}{r} \right]^2 \right\} \Bigg|_{r=r_i} \tag{29}$$

For low energies of injection, the second quantity in braces is by equation (15), the voltage of injection. The first braced quantity is a geometrical factor which depends both on the field shape of the betatron and the choice of C. From Figs. 1 and 2, the value of this factor for the optimum C of $-.008\ P_{1s}$ is approximately $\frac{1}{4}$ but for a C of $+.1\ P_{1s}$ it is about 1/100. Excepting for the geometrical factor, equation (29) is the same as that of Kerst (9).

The maximum supportable charge, and hence the maximum space charge limited output of the betatron, is, other things being equal, directly proportional to the injection voltage. It has been quite generally observed that the output increases with injection voltage, but the total output has been of the order of ten per cent., or less, of that computed by the old formula or what is practically the same thing, formula (29) with the optimum geometrical factor. The injection voltages currently in use seem too high for electron scattering by residual gas to enter into this effect.

The implication of these experimental results appears to be that while the betatron may at injection approach a space charge limited condition, the injected electron groups which can be captured in stable orbits are those of tangential velocities or C values departing considerably from the optimum.

MOTION IN THE POTENTIAL OF FORCES.

The increase of the magnetic field with time brings about the acceleration of the electrons and the change in magnitude and shape of the appropriate potential of forces $[V_M - V_E]$. While the energy given by equation (15) in combination with (19) increases at first with the square of $r B_z$ at the equilibrium orbit, and for relativistic velocities directly with $r B_z$, the radial and axial oscillations about the orbit are altered by the change in the potential surface. The existence of a minimum determines the possibility of stable electron orbits, but during the acceleration period the limits and velocities of the oscillations are changed. On this account, the electrons might alternatively strike the walls of the vacuum chamber and injection apparatus or be focused into a narrow beam.

The radial and axial motion constitutes two-dimensional motion in a potential field changing with time, but most of the essential characteristics can be found in a corresponding one-dimensional motion. This is particularly true in the conventional betatron because it can be shown by a Taylor expansion of the potential V_M near the main equilibrium orbit that the motions in the r and z directions are independent up to fourth order terms. The potential surface changes but little in the period of one oscillation of the particle. An approximation in the analysis will be made based on the smallness of the change in energy per oscillation as compared to the energy itself, namely, an adiabatic approximation.

Consider at first the equation of a constant mass point in a potential of forces W, of the form $W(x, t) = G(x)F(t)$ where $G(x)$ is a non-negative function of space only and $F(t)$ a slowly varying function of time only:

$$\ddot{x} = -\frac{\partial W}{\partial x}. \tag{30}$$

Without ultimate loss in generality, it may be assumed for simplicity that $G(0) = 0$. An energy integral can be obtained by integrating both sides of equation (30) multiplied by $\dot{x}$:

$$\tfrac{1}{2}(\dot{x}^2 - \dot{x}_0^2) = -W + W(x_0, t_0) + \int_{t_0}^{t} \frac{\partial W}{\partial t}\, dt, \tag{31}$$

where $x = x_0$ and $\dot{x} = \dot{x}_0$ at $t = t_0$. The value of the integral in this equation can be estimated by the law of the mean since $G > 0$.

$$\int_{t_0}^{t} \frac{\partial W}{\partial t}\, dt = \int_{t_0}^{t} G \frac{dF}{dt}\, dt = \left[\frac{1}{F}\frac{dF}{dt}\right]_{t_1} \int_{t_0}^{t} W(x, t)dt,$$

where $t_0 < t_1 < t$. (32)

By substituting W from equation (31) into the last integral, $\int_{t_0}^{t} \frac{\partial W}{\partial t}\, dt$

can be written:

$$\int_{t_0}^{t} \frac{\partial W}{\partial t}\, dt = (1 + \epsilon)\left[\frac{1}{F}\frac{dF}{dt}\right]_{t_1} \int_{t_0}^{t}\left[W(x_0, t_0) + \frac{\dot{x}_0^2 - \dot{x}^2}{2}\right] dt, \quad (33)$$

where

$$|\epsilon| < \tfrac{1}{2}\left[\frac{1}{F}\frac{dF}{dt}\right]_{\max} (t - t_0)\, \frac{W_{\max}}{W_0 + \dfrac{\dot{x}_0^2}{2} - \dfrac{1}{t - t_0}\displaystyle\int_{t_0}^{t} \frac{\dot{x}^2}{2}\, dt}. \quad (34)$$

The value of $\frac{1}{F}\frac{dF}{dt}(t - t_0)$ is the proportional change of energy of oscillations per period and is assumed very small in the adiabatic approximation. Therefore, the value of ϵ is also small since the ratio of the maximum value of the potential in the interval to the difference between the initial energy and the average kinetic energy is of the order of unity. In the application of this idealized problem to the present day betatrons, ϵ is less than one per cent. and will be neglected. By substituting into equation (31) the value from equation (33) where $\frac{1}{F}\frac{dF}{dt}$ at t_1 is assumed equal to its value at t_0, again according to the adiabatic approximation, the total energy of oscillation E of the mass point becomes:

$$E = W + \frac{\dot{x}^2}{2} = W(x_0, t_0) + \frac{\dot{x}_0^2}{2} + \tfrac{1}{2}\frac{1}{F(t_0)}\frac{dF}{dt}\left[\left(W(x_0, t_0) + \frac{\dot{x}_0^2}{2}\right)(t - t_0) - \int_{t_0}^{t} \frac{\dot{x}^2}{2}\, dt\right]. \quad (35)$$

Let us assume an oscillatory motion such that at $t = t_0$, $\dot{x}_0 = 0$ and $x = a$ and again $\dot{x} = 0$ when $t = t_0 + T$ at which time $x = a + \Delta a$. The increase of energy ΔE during the period T is:

$$\Delta E = \frac{1}{F(t_0)}\frac{dF}{dt}\left[W(x_0, t_0)\, T - \int_{t_0}^{t_0+T} \frac{\dot{x}^2}{2}\, dt\right]$$

$$= \Delta W - \frac{\Delta W}{W(x_0, t_0)}\,\frac{1}{T}\int_{t_0}^{t_0+T} \frac{\dot{x}^2}{2}\, dt,$$

$$\Delta E = \Delta W - \delta E =$$

$$\Delta W\left(1 - \frac{\text{Average kinetic energy in one period}}{\text{Maximum kinetic energy in period}}\right) \quad (36)$$

where ΔW is the increase of W at the point $x = a$ and δE is what may be called the energy increment deficiency because it is the difference between the most energy which the mass point would gain were it immobilized at $x = a$ and its actual gain of energy while oscillating in the increasing potential trough.

The change of amplitude of oscillation, Δa, is evidently related to the energy increment deficiency by the approximate relation:

$$\Delta a = \frac{\delta E}{\left.\frac{\partial W}{\partial x}\right|_{x=a}} = -\frac{\text{Average kinetic energy}}{\text{Maximum kinetic energy}} \frac{\Delta W}{\left.\frac{\partial W}{\partial x}\right|_{x=a}}. \tag{37}$$

showing that it is proportional to the rate of change of the potential, the ratio of the average to maximum kinetic energy and inversely proportional to the gradient of the potential at the turning point of zero kinetic energy.

In the case of harmonic motion in a parabolic potential, the average kinetic energy is half of the maximum and the gradient at the extremum is twice the average gradient from the minimum to the extremum, so that:

$$\frac{\Delta a}{a} = -\tfrac{1}{4}\frac{\Delta W}{W}. \tag{38}$$

These considerations can be applied to the actual case of the electron in the V_M potential surface by writing:

$$W = \frac{e}{m_0}\left[V_M(r, t) - V_M(r_c, t)\right]. \tag{39}$$

Relation (38) can then be written, for the normal electrons, $C = 0$, as:

$$\frac{\Delta a}{a} = -\tfrac{1}{4}\frac{\Delta V_M}{V_M} = -\tfrac{1}{2}\frac{\Delta B}{B}, \tag{40}$$

relations derived by Kerst and Serber (10). These relations are valid even when the terms involving dm/dt which were omitted in deriving equation (14) are taken into account. This relation (40) shows that the amplitude of oscillation is proportional to $B^{-\frac{1}{2}}$.

In the case of the parabolic potential, the integrated equation of motion itself of the mass point can be obtained and a closer estimate can be made of the error due to the adiabatic approximation than was possible in the general derivation. Equation (30) takes the form $\ddot{x} = -x(\partial^2 W/\partial x^2)$ where $\partial^2 W/\partial x^2$ is a function of time only, and can be solved approximately by the substitution $x = u\,e^v$ which leads to:

$$x = A\left(\frac{\partial^2 W}{\partial x^2}\right)^{-\frac{1}{4}} \cos\left(\int_{t_0}^{t}\sqrt{\frac{\partial^2 W}{\partial x^2}}\,dt + \alpha\right), \tag{41}$$

where the amplitude of oscillation is seen to vary as found before and the instantaneous angular frequency of oscillation is $\sqrt{\frac{\partial^2 W}{\partial x^2}}$. A differential equation can now be written which is exactly satisfied by equation (41) whose form is $\ddot{x} + x\frac{\partial^2 W}{\partial x^2}(1 + \epsilon_1) = 0$. The proportional

error ϵ_1 with respect to equation (30) can be computed:

$$\epsilon_1 = \left(\frac{\partial^2 W}{\partial x^2}\right)^{-2}\left[\frac{1}{4}\frac{\partial^2}{\partial t^2}\left(\frac{\partial^2 W}{\partial x^2}\right) - \frac{5}{16}\left(\frac{\partial^2 W}{\partial x^2}\right)^{-1}\left(\frac{\partial}{\partial t}\frac{\partial^2 W}{\partial x^2}\right)^2\right]. \tag{42}$$

For the case of non-relativistic speeds of the corresponding electron motion, if $\nu = \frac{e}{m} B_e$ is the instantaneous Larmor frequency corresponding to the field B_e, this error is less than:

$$\epsilon_1 < \frac{\omega^2}{\nu^2}\frac{B^2_m}{B^2_e}\frac{1}{1-n}\left(\tfrac{1}{2} + \tfrac{1}{4}\cos^2 \omega t\right), \tag{43}$$

assuming the magnetic field to vary sinusoidally as $B_e = B_m \sin \omega t$. The error is most at injection time but is extremely small, of the order of 10^{-10} to 10^{-8} for practical instruments, since it depends on the square of the ratio of the magnet and electron Larmor frequencies as mentioned by Kerst and Serber (10).

In general, the potential function V_M or $[V_M - V_E]$ cannot be represented as a product of separate space and time functions, particularly because C appears in V_M. The position of the minimum changes with time but the oscillations of the electrons will take place around the instantaneous position in a reference system moving with it, just as if the minimum position were stationary. This is because the rate of acceleration of that position is negligible in all practical cases. Since equation (19) implicitly expresses the radial position of this minimum, r_i, for the conventional betatron, it may be used to give $\dot{r}_i$ through differentiation.

$$\dot{r}_i = \frac{\dfrac{\int_0^r r\dfrac{\partial B_z}{\partial t}\,dr}{r^2} - \dfrac{\partial B_z}{\partial t}}{\dfrac{B_z}{r}\left[\dfrac{r}{B_z}\dfrac{\partial B_z}{\partial r} + 1\right]}. \quad \text{at } r = r_i. \tag{44}$$

Assuming the magnetic flux and field to increase synchronously, this equation may be approximated for very small values of the difference $r_i - r_e = x_i$ and written for a small time increment Δt.

$$\frac{\Delta x_i}{x_i} = -\left.\frac{\Delta B_z}{B_z}\right|_{r=r_e}. \tag{45}$$

For non-relativistic energies, this is equivalent by equation (15) to a relation obtained by Kerst and Serber (10).

$$\frac{\Delta x_i}{x_i} = -\tfrac{1}{2}\frac{\Delta V}{V}. \tag{46}$$

Thus the instantaneous equilibrium orbit shifts asymptotically toward the main equilibrium orbit so that the separation is at first inversely proportional to the magnetic field. This shift is in contrast to the damping of the amplitude of oscillation which is inversely proportional to the square root of the magnetic field.

An important departure of the electron motion from what has already been described may take place during the latter part of the acceleration period from force terms which have hitherto been neglected in the equations of motion. Iwanenko and Pomeranchuk (19) have pointed out that in consequence of the high centripetal acceleration of the electron in its orbit, the charged particle will radiate energy at a rate which for large values of its own kinetic energy (of the order of 100 Mev) may be a considerable proportion of the rate of energy gain from the induction field. The results of this will be first a change in the position of the equilibrium orbit and ultimately a limit on the attainable kinetic energy. This limitation can be offset, in principle, by building high energy accelerators with relatively smaller centripetal accelerations, that is, with smaller field strengths and larger orbital radii, or with shorter acceleration periods, that is, higher frequency (12). If the amount of energy radiated were known, the effects on the electron path could be computed from present formulas with the aid of equation (15) by introducing an artificial time variation to the constant C. However, it is at present not clear whether the radiation formulas for a single electron are immediately applicable to the case of the multitude of circulating electrons in the betatron. References have been made in the literature to a paper by J. S. Schwinger (39) which is shortly to appear and which presumably will give a definitive treatment of the radiation problem. See also ref. 28.

INJECTION IN A PURELY MAGNETIC BETATRON.

The betatron has been discussed thus far under the assumption of perfect symmetry of mechanical arrangement, and electrons were considered present with appropriate energies without regard to any actual injection mechanism. In the conventional magnetic betatron electrons are injected circumferentially from a gun of finite dimensions, which is necessarily located in the region of acceleration of the particles. The gun presents, therefore, a possible obstruction to their manifold revolutions. In fact, it is found experimentally that the output of the betatron is very much less than the upper limit imposed by space charge.

Upon injection, the electrons oscillate, as has been shown, around instantaneous equilibrium orbits shifting radially in the plane $z = 0$, with axial and radial amplitudes which are damped as the energy increases. Since the extremum positions of oscillations recede in the radial direction on account of both the change in amplitude and the

shift of equilibrium orbit, while axially only the change in amplitude is effective, it is more favorable to locate the center of the gun at some radial distance x_0 from the main equilibrium orbit r_e rather than away from the plane $z = 0$. An electron will return to the gun, or to a region nearby where the electrostatic leakage field may cause it to strike the gun, at some time T after emission. This time depends on the relation between the period of revolution T_θ and the period of radial oscillation T_r, in connection with the angular and radial extents of the gun. For an exactly tangential direction of injection, a particular electron will succeed in missing the gun if the total recession of the path δr, in the times T of the first or any other subsequent passage very near the gun, is greater than the distance d between the point of injection and the edge of the region near the gun where collision can occur. If, at injection, the electron has a small radial velocity in addition, the situation is less favorable but still depends on the magnitude of the recession. This total recession δr is evidently the sum of the decrements $(\Delta r_i + \Delta a)$ occurring in time T, and is obtained from equations (37) and (45) where ΔV, the energy gain in this time, is approximately the product of the number of revolutions N and the energy gain per revolution, w "volts per turn."

If the radial section of the potential trough is approximated by a parabola, which is a consideration essentially the same as the first order treatment of Kerst and Serber (10), the total recession δr immediately after injection is given by equations (38) and (46).

$$\delta r = -\tfrac{1}{2}(x_{i0} + \tfrac{1}{2}a_0)\frac{Nw}{V_i}. \tag{47}$$

Since the sum of the initial instantaneous orbit position x_{i0} and the initial amplitude a_0 is the total initial distance x_0 from the main equilibrium orbit, the condition for avoiding the gun requires at least that

$$\tfrac{1}{2}Nwx_0 > V_i d. \tag{48}$$

The value of N can be estimated from the sinusoidal nature of the trajectory by considering the ratio T_r/T_θ from equations (24) and (25) in relation to the mechanical dimensions of the gun. For an experimental arrangement used by the present authors, the value of N varied from 2 to 6, depending on the injection constant C but always within the approximation that the potential functions are merely first order departures from the main equilibrium radius. In general practice it is found experimentally that much higher values of the product $V_i d$ than those of the above inequality (48) can be used. Indeed, the output of the betatron apparently increases with the voltage of injection V_i. It may be expected, however, that the actual non-parabolic nature of the potentials for some injection parameters will materially alter the

factors contributing to the recession δr, these factors being the damping of the amplitude of oscillation Δa, the equilibrium orbit shift Δx_i, and the period T of consecutive passages very near the gun.

As has already been pointed out, the range of C values of electrons which can be captured in stable orbits depends first on the existence of a minimum in the potential function and second, that the walls of the bowl are everywhere as high as at the point of injection. In conventional practice, all electrons are injected at the same voltage throughout an interval of time, so that the range of C values comes about through differences in the time of injection, and hence in the value of the magnetic field integral appearing in C, equation (8).

For values of C toward the ends of this range of capturable electrons, the radial sections of the potential surfaces depart significantly from the parabolic approximation. When the gun is located further from the main equilibrium orbit than the extrema of the locus of extrema ($r_0 < 8.5$ and $r_0 > 17.5$ for the example of Fig. 2), there is a value C_1 of C for which the potential V_M has a maximum at the gun. The rate of change of the amplitude of oscillations of electrons injected at times just following that corresponding to C_1 (when $r_0 > 17.5$, or just preceding when $r_0 < 8.5$), is very large since it is inversely proportional to the value of $\partial V_M/\partial r$ at the injection point in accordance with equation (37). Since this rate becomes very large for injection times corresponding to C_1, the gun clearance may be improved by a large factor of the order of a hundred over that of the parabolic case, provided of course that the gun is so located that the electrons are not lost in their first swing across the equilibrium orbit on the side opposite to the gun. ($7 < r_0 < 8.5$ or $17.5 < r_0 < 22.5$, in the example of Fig. 2.) The recession will depend on the product of the rate of change of amplitude by the time T of consecutive passages near the gun, and therefore on the number N of revolutions between dangerously close approaches. The number N varies with C and passes through small and large values depending on the ratio of T_r/T_θ. Hence there is the possibility of capturing electrons in several distinct groups corresponding to the large values of N. The capture of electrons through the large damping of amplitude will occur in a range of C values close to C_1 (smaller than C_1 for an external gun, greater for an internal one), which is a comparatively small part of the total range corresponding to potentials with a minimum and in general the potential trough will be shallower than the normal curve ($C = 0$) with hence less space charge capacity.

Consider the gun located at or near the limit at which the potential can have a maximum, that is, at or near one of the extrema of the locus of extrema of the potential function ($r_0 = 8.5$ and $r_0 = 17.5$ in Fig. 2). At these points there will be a value C_2 of C for which the

potential V_M will have an inflection point. Near such a point the rate of shifting of the instantaneous equilibrium orbit, which is inversely proportional to $\left(1 + \frac{r}{B_z}\frac{\partial B_z}{\partial r}\right)$ as shown by equation (44), becomes very large. Since it approaches infinite values at the point itself, here again the position of the electrons may recede in the time T, which is at least equal to T_θ, an amount δr greater than d even if the amplitude damping were not taken into account. The range of C values for which the rate of equilibrium orbit shift is sufficient is only a small part of the range for which equilibrium orbits exist. Furthermore, the potential troughs are very shallow and it is probable that the number of electrons captured in them approaches being space charge limited, even for small injecting currents.

Finally, consider the case of an injector located at a position such that for some value C_3 of C the initial potential curve has a maximum whose value is equal to that at the injector position but which is located on the opposite side of the potential minimum. It is apparent that for C values approaching C_3, greater for an external gun and smaller for an internal one, the period of oscillation T_r around the equilibrium orbit tends to infinity since the radial velocity of the electron near the maximum of V is very small. Consequently the time T becomes very large and for a finite rate of shifting of the equilibrium orbit given by equation (44), the recession δr of the electrons may be sufficient for clearing the gun, provided the shift of the orbit is away and not toward the gun. This occurs only for particular gun positions and C values (for example $r_0 = 8$, $C = -.004$) which are small compared to the total range having minima in the potential.

These considerations show that for high injection voltages the principal mechanism of capture is the damping of amplitude of oscillation when the potential function has a maximum near the gun. The cases of rapid shift of the equilibrium orbit and of long periods of radial oscillation may account for similar capture but are less probable mechanisms.

To estimate the interval of time over which electrons can be captured, consider a synchronously varying magnetic field, or one which can be expressed as the product of separate functions of space and time:

$$B(r, z, t) = b(r, z) \cdot f(t). \tag{49}$$

The potential V can then be written as:

$$V_M = \frac{em_0}{2m^2} f^2(t) \left[\frac{\int_0^r brdr + \frac{f(t_0)}{f(t)} D}{r} \right]^2, \tag{50}$$

where t_0 is the injection time and $D = C/f(t_0)$. This relation shows

explicitly as has been implicitly assumed so far, that the scale factor of the potential surface after emission time increases in the ratio $f^2(t)/f^2(t_0)$ while the shape of the surface changes with respect to one derivable from an unchanging magnetic field just as if the constant C decreased in the ratio $f(t_0)/f(t)$. The interval of time t_1 to t_2 in which the C values correspond to potentials with a minimum is therefore, for the case illustrated in Fig. 2:

$$\frac{f(t_2)}{f(t_1)} = \frac{P_{18} + C_{\max}}{P_{18} + C_{\min}} = 1.15, \tag{51}$$

corresponding approximately to 15 per cent. of the phase angle of the injection period characteristic of a sinusoidally varying magnetic field, an angle proportional to the square root of the injection voltage. The range of C values corresponding to initial functions having the special properties necessary for capture is a small part of the total range of C values characterizing a minimum of the potentials V_M and can be estimated on the basis of the size and location of the gun and is unlikely to exceed 10 per cent. in practical cases. The effective phase in which electrons can be captured is therefore only a small percentage, about 1 per cent., of the injection phase itself, and the corresponding effective time of capture is quite short (about 10^{-7} second for 4,000 volts injection voltage and other parameters used in the example of this paper). In spite of this, it is found that in present day betatrons the current emission capabilities of conventional guns are ample to provide a charge equal to the limit given by space charge, equation (29).

The mechanism of gun clearing was accounted for by ignoring the effects of space charge, while the upper limit of capturable charge was estimated in a preceding section by ignoring the obstruction of the gun. The actual effects are only partially described by these two limiting, idealized cases. However, it is probable that even if the capturing potential troughs were space charge limited, this fact would not alter the existence of limiting requirements on values for the damping of the amplitude of oscillation, the rate of equilibrium orbit shift or the period T of consecutive passages near the gun, which are primarily responsible for the gun clearing mechanism. It is even possible that space charge has an enhancing effect in damping oscillations for if one supposes formula (37) to be extended to such a case, the principal alteration would be a reduction in $\partial V_M/\partial r$, with a consequent considerable increase in Δa. On the other hand, it is very likely that the particularities required from the potential function to avoid the gun obstruction are inconsistent with large capturable charges and are responsible for the low percentages of the upper space charge limit which are usually observed in actual experiments.

(End of Part I)

Industrial X-Ray. (*Scientific American*, Vol. 175, No. 5.)—Installation of a large X-ray machine—2,000,000 volts—for the purpose of examining welds in pressure vessels will bring to 12 the number of X-ray machines used for this purpose by the Babcock and Wilcox Company, manufacturers of steam generating equipment. A special building will house the new machine and it will be erected adjacent to the location of the present 1,000,000-volt X-ray unit. The building has been designed to protect personnel from radiation and to make possible examination of the largest size drums made by the company.

Although the apparatus, which was purchased from the General Electric X-Ray Corporation, is capable of operating at any voltage from 1,000,000 to 2,000,000, it will be used principally at the higher voltages to take pictures of heavy welds, four inches thick and over. As compared to the 1,000,000-volt X-ray, the new unit will produce much clearer pictures in that thickness, and will reduce by half the average time required to take radiographs. It can be used successfully on steel walls 10 inches thick. The specially designed building will be 35 feet wide, 31 feet high, and 81 feet in length. Double walls consisting of 42 inches of sand solidly packed between quarter-inch steel plates will prevent radiation from the unit. The building is planned to provide all possible safeguards to protect not only technicians within the building, but anyone in the immediate area.

An overhead, floor-operated crane will support the X-ray unit and move it around within the building. The X-ray machine is 5 feet in diameter, 12½ feet in length, and weighs 6,000 pounds.

The building will accommodate drums up to 13 feet in diameter and 75 feet in length, which are the dimensions of the largest vessel likely to be shipped by rail. Vessels that weigh up to 150 tons can be tested. This probably represents the maximum weight of pressure vessels that the firm might be called upon to make the test. Two cars with drum turners will bring the drums into the building. One car and one drum turner will be motorized to facilitate handling.

R. H. OPPERMANN.

ATMOSPHERIC ELECTRICITY AND LIGHTNING.*

BY

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CHAPTER I. FUNDAMENTAL EMPIRICAL DATA.

1. Electrical Nature of Lightning and Thunder.

The phenomena characteristic of thunder storms, like many other common phenomena, familiar to humanity from ancient times, remained wholly unintelligible until the development of physics led to the discovery and experimental investigation of similar phenomena under laboratory conditions and until this investigation disclosed the similarity, masked by the difference of scale, between the miniature laboratory model and the huge natural original. Only in the XVIII century did it flash through the mind of the scientists that the spark, rushing between two oppositely electrified bodies, is the analogon of lightning, while the accompanying sound—the analogon of thunder. This electrical theory of thunder storms was definitely established by Franklin. In spite of the tremendous success in the subsequent development of electrical theory in general and in the study of the phenomena of atmospheric electricity in particular, the mechanism of these phenomena has not been fully understood up to the present time.

2. Atmospheric Electric Field under Fair Weather Conditions.

One of the most important steps in the development of our knowledge of the phenomena of atmospheric electricity was the discovery of the fact that an electric field exists in the atmosphere not only in the region of thunder storms, but likewise under a wholly clear sky. In the latter case this "normal" electrical field is directed downwards; near the surface of the ground it reaches its highest value of the order of 1 v./cm., and gradually decreases with increase of altitude, practically vanishing at an altitude of 10–12 km. (where its distribution in space has not yet been studied accurately).

During a thunder storm the electrical field over the ground has, as a rule, a direction opposite to the normal one and, on the average, a hundredfold larger magnitude.

The structure of the normal electrical field indicates that in the absence of clouds the surface of the ground bears a negative charge, while the atmosphere, an equivalent positive charge, distributed more

* Author's translation of a Russian paper, presented at the General Meeting of the U.S.S.R. Academy of Sciences on July 2nd, 1946.

or less uniformly in its lower layer, called the troposphere. If the same conditions prevailed in the clouded regions, the terrestrial globe could be regarded as a huge spherical condenser with an inner negative surface charge on the ground and an outer positive space charge in the troposphere.

3. The Electrical Field of the Clouds.

The investigations of the last 10–15 years have shown, however, that this picture does not correspond to reality. As has already been mentioned above, the electrical field under thunder clouds is much more intense than the normal one and has the opposite sign, i.e. is directed upwards. It preserves, as a rule, the same negative direction below ordinary cumulus clouds, while its magnitude is in this case smaller. Thus, in the clouded regions the surface of the ground is, as a rule, positively charged, while the lower surface of these clouds bears a negative charge.

The investigations of the outstanding British scientist Sir G. Simpson, carried out a few years before the war, have shown that the upper part of the clouds—both thunder clouds and partially ordinary ones—is positively charged, so that within the cloud the electrical field is directed downwards and above the cloud—upwards, i.e. in the same way as below it (Fig. 1). Simpson has further established the sur-

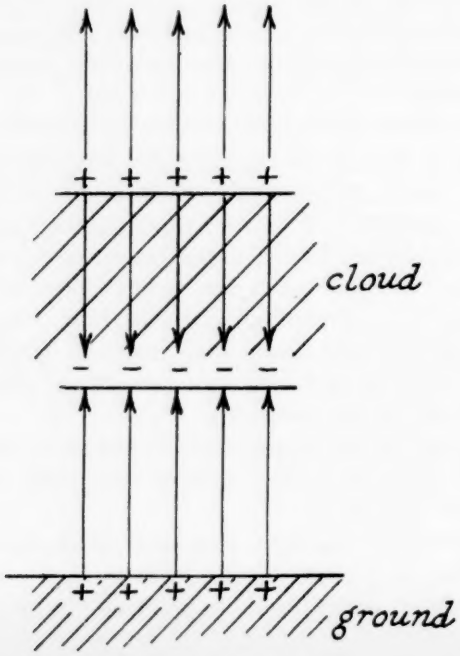

FIG. 1.

prising fact that the average strength of the electrical field within the clouds is of the same order of magnitude—namely of the order of 120 v./cm.—both in thunderclouds and in ordinary clouds (in spite of the fact that the initiation of sparking discharges requires a field strength of the order of 10,000 v./cm.

Hence it follows that the excessively high field strengths which cause lightning discharges in thunder clouds are localized within relatively small regions of these clouds. This circumstance is in agreement with the modern conception on the mechanism of the propagation of a lightning discharge: after its initiation the latter can be propagated in the shape of a conducting channel with a gradually increasing length in a relatively weak electrical field.

4. Ionization and Electrical Conductivity of the Lower Atmosphere.

The atmospheric air is well known to possess a very small electrical conductivity, due to the presence of positive and negative ions in equivalent amounts of the order of 1,000 per cc. (near the ground). The ionization of the lower atmosphere is mainly produced by the action of cosmic rays, and to some extent, near the surface of the ground, by radioactive substances, emanating from the latter. The intensity of the cosmic rays decreases with their penetration through the atmosphere; since, on the other hand, the density of the air increases, the number of ions (or ion-pairs) per unit volume, and consequently the electric conductivity of the lower atmosphere, first increases with approach to the surface of the ground, reaching a maximum at an altitude of 18 km., and thereafter steadily decreases. Near the ground, at sea-level, it is about 30 times smaller than at an altitude of 10–12 km.

It should be noticed that similar ionization maxima are found in the higher atmosphere, at an altitude of 100 km. (*E* level) and about 400 km. (*F* level); these high altitude ionization maxima are, however, due to the ultra-violet radiation of the sun, which is absorbed in the atmosphere much more rapidly than the cosmic radiation, reaching the earth from beyond the boundaries of the solar system. This also explains the fact that the ionization of the lower atmosphere exhibits but a relatively insignificant diurnal variation due to secondary factors (such as the amount of dust or smoke in the air).

Owing to the electrical conductivity of air, the electrical charges distributed on the surface of the ground should be "dissipated" in the same way as the electric charge of any electrified body in contact with air, in the absence of an external source of electricity, i.e. an electric generator that would compensate for the loss of electricity due to dissipation.

5. Electric Polarization of the Troposphere and Depolarizing Electric Currents.

The notion of "electrification" and "dissipation of electric charge" conveys a somewhat misleading conception as to the mechanism of the corresponding processes. The electrification of certain bodies, constituting the system under consideration, always implies the electrification of some other bodies with an equivalent charge of the opposite sign. If all the bodies are contemplated as a single system, one must speak not about electrification (since the total charge remains constant), but about a *separation of electrical charges of opposite sign in space*. This process is usually denoted as *electric polarization* of the corresponding system. The opposite process of the recombination of electrical charges of opposite sign, or *depolarization*, leads to their apparent dissipation. This dissipation process means, accordingly, a motion of positive charges towards negative ones or negative charges towards positive, or both. Such a motion of electrical charges constitutes an *electric current*. The direction of the latter coincides with the direction of the electric lines of force, and its strength or, rather, density,[1] is expressed as the product of the electric field strength by the specific conductivity of the respective medium, that is, in our case, of air. Under a clear sky the electric current, at small altitudes at least, flows downwards—from the positively charged air to the negatively charged surface of the ground, while under thunder clouds (or other clouds of a similar nature) it flows upwards from the surface of the ground to the lower edge of the clouds.

Starting from the value of the electrical conductivity ($\lambda = 2.10^{-4}$ E.S.U.), it can easily be calculated that this "depolarizing" or "discharge" current would destroy that spatial separation of positive and negative charges to which the electrical field generating this current is due, in a time interval of the order of 10–20 minutes.[2] There arises thus the question of the mechanism, ensuring the persistence of the atmospheric electric field—in other words, of the nature of the electric generator, which is responsible for the persistence of the electrical polarization of the system, constituted by the surface of the ground and the lower atmosphere.

CHAPTER II. INITIAL SEPARATION OF ELECTRICAL CHARGES IN THE ATMOSPHERE AS A RESULT OF THE IONIZATION OF THE AIR AND THE NEGATIVE ELECTRIFICATION OF CLOUD PARTICLES.

6. The Cloud as a Colloidal System and the Negative Electrification of the Particles Constituting it.

A cloud must be regarded as a disperse system, consisting of small water drops or ice-crystals, which are slowly sinking with respect to air, the latter playing the rôle of the disperse medium.

[1] I.e. the amount of electricity passing per unit cross section in unit time.

[2] This time is of the order of $\frac{1}{4\pi\lambda}$.

In the presence of positive and negative ions (to which the electric conductivity of the air is due), these cloud particles become charged, either positively or negatively, just as in the case of an ordinary colloidal solution in the presence of ions of both signs in the disperse medium (aqueous solution of an electrolyte). The colloidal particles usually attract ions of one sign stronger than those of the opposite sign, and acquire, as a result, an electric charge of a definite sign, corresponding to a fixed value of the electrokinetic potential, while the disperse medium acquires a space charge of the opposite sign. If the latter has a large density, it concentrates within a thin layer surrounding the surface of the particles of the disperse phase. In the contrary case it is distributed more or less uniformly in the volume of the disperse medium.

It has been established experimentally that in an atmosphere of ionized air water drops become negatively charged.[3] This fact is apparently connected with the polarity of water, i.e. with the permanent polarization of the individual molecules, along with their partial orientation in the surface layer of a water drop, with their negative ends pointing outwards. This orientation is revealed experimentally by the existence of a potential jump approximately equal to 0.25 volt in the plane surface layer of water in bulk (according to Langmuir and Frumkin). Under such conditions, the capture by a neutral water drop of negative ions from the surrounding air leads to a greater decrease of energy than the capture of positive ions. In a state of statistical equilibrium the water drops must acquire an electric potential V, equal and opposite to the potential jump in the surface layer of water, that is a potential of -0.25 volt (a subsequent capture of the ions of both signs becomes equally advantageous from the point of view of energy). The corresponding equilibrium charge of the drops must thus be proportional to their radius r, i.e. equal to $q = rV$. It seems natural to expect that this relation remains valid in the case of small ice-crystals with linear dimensions of the same order of magnitude $r = 10^{-3}$ cm., as that characteristic of the water drops, constituting the main bulk of a cloud.

7. Positive Electrification of Large Water Drops under the Influence of the Atmospheric Electric Field.

In the case of larger water particles, sinking down in the form of rain drops (or snow-flakes), the preceding relation becomes complicated by the polarizing (or "induction") effect, due to the influence of the electric field E of the atmosphere. This influence consists in the opposite electrification of the lower and higher hemispheres of the drop—

[3] This phenomenon has originally been misinterpreted as a result of the condensation of supersaturated vapour on negative ions. Actually the embryonal water drops arise without the participation of ions. The rôle of the latter consists in the stabilization of water drops, to which they become attached (irrespective of their electric charge), i.e. to the reduction of their vapour pressure and the resulting decrease of the rate of their evaporation, which enables them to reach a size when their growth in the supersaturated vapour can proceed indefinitely.

if the electric field has a vertical direction. Now, the negatively charged hemisphere attracts positive ions, while the positive one—the negative ones. As a result of the capture of the attracted ions by the corresponding hemispheres, the induced charges are decreased but *to a different extent*, those ions which contribute a larger part to the total conductivity of air λ being more effective in this respect.

Now, it is known experimentally that the contribution of the positive ions λ_+ is larger than that of the negative ones λ_-. This fact can be considered as a consequence of the larger affinity of negative ions to the water drops suspended in the atmosphere; as a result, the concentration of the *free* positive ions is larger than that of the negative ones, so much larger indeed that, in spite of their lower mobility, the positive ions give a larger contribution to the electrical conductivity of the air (the ratio λ_+/λ_- is approximately equal to 1.3).

The flow of positive ions to the negative hemisphere of the polarized drop must therefore decrease its charge at a somewhat higher rate than the flow of the negative ions to the positive hemisphere. The drop, as a whole, must acquire an additional positive charge. It can easily be shown that this charge, due to the induction effect of the atmospheric electric field, is proportional to the field strength E and to the square of the radius of the drop.[4] So long as the drop is sufficiently small, this additional charge decreases insignificantly its negative charge which is due to the larger affinity with respect to the negative ions, and which is proportional to the first power of the radius. If, however, the latter exceeds a certain critical value (of the order of 0.02 cm. in the case of a field with a strength 1 v./cm.), the "additional" positive charge becomes larger than the "fundamental" negative one, so that the drop acquires a resulting positive charge.

The preceding considerations enable one to explain the fact that the rain drops falling on the surface of the ground, with a radius of the order of 1–2 mm., carry, as a rule, a positive charge. During their fall, lasting several minutes, these drops have a fair chance of altering the sign of their charge from negative to positive, especially in the abnormally strong electric field prevailing within the cloud and beneath it.

CHAPTER III. THE CLOUDS AS GENERATORS OF THE ELECTRICAL FIELD AND OF THE ELECTRICAL CIRCULATION IN THE ATMOSPHERE AND THE GROUND.

8. Electrical Polarization of a Cloud under the Influence of Gravity.

The positive electrification of large water drops (rain drops) does not in the least contradict our original assumption that the small drops with a radius of the order of 0.01 mm., constituting the bulk of a cloud, are on the average charged negatively.[5] We shall now show that this

[4] Being of the order $\frac{\lambda_+ - \lambda_-}{\lambda_+ + \lambda_-} r^2 E$.

[5] Actually the magnitude and even the sign of their charge can fluctuate within pretty wide limits owing to thermal motion.

assumption, along with the underlying fact of the ionization of the lower atmosphere, gives a key to the understanding of the mechanism of the generation and preservation of the atmospheric electric field.

The negatively charged cloud particles must sink with respect to the surrounding air with a velocity of the order of a few millimeters per second. As regards the free ions remaining in the air, they are practically uninfluenced by the direct action of gravity. If, however, they remained fixed with respect to the air while the cloud particles were sinking down, the positive charge of the air, due to the excess of the free positive ions over the negative ones, should become more and more separated from the negative charge associated with the drops.

Let us imagine, for example, that the cloud has a spherical shape and that its two constituent parts—the negatively charged sphere consisting of the negatively charged water drops and the positively charged sphere, consisting of the air with its excess of free positive ions—initially coincide with each other. After a certain time t the negative sphere will sink below the positive one at a distance vt, where v is the velocity of sinking (with respect to air; the motion of the air as a whole, along with the cloud particles, is wholly immaterial for the process under consideration). The space between the two lower hemispheres will be filled with negative electricity, while the space between the two upper hemispheres—with an equivalent amount of positive electricity (see Fig. 4). The force of gravity thus produces an electrical polarization of the cloud. This polarization gives rise to a macroscopic electric field whose lines of force start on the positive charges appearing on the upper edge of the cloud, and terminate on the negative charges of its lower edge. It should be noticed that inside the cloud these lines are directed vertically downwards, while outside it they have the same direction near the equatorial plane and an opposite—upward—direction near the axis.

If the ions were wholly deprived of mobility, i.e. were fixed with respect to the air, the cloud particles would sink until their weight is balanced by the electrical force produced by the polarization of the cloud. The corresponding field strength E is determined by the equation $qE = mg$, where q is the charge of a cloud particle, m—its mass, and g—the acceleration of gravity. Putting $q = Vr$, where V is the potential of a cloud particle, and $m = \frac{4\pi}{3} r^3$ (since the specific weight of water is equal to 1), we obtain for this field the expression $E = \frac{4\pi}{3} \frac{gr^2}{V}$. which with $V = 10^{-3}$ E.S.U. ($\cong 0.3$ volt) and $r = 10^{-3}$ cm. gives $E = 4$ E.S.U., that is, 1,200 volt/cm.

In reality, however, the ions are *not* fixed with respect to the air, and can move relative to it under the influence of electrical forces. The electrical field, due to the polarization of the cloud, brings the ions into

motion—not only in the region near the upper edge of the cloud, where there exists an excess of positive ions, but also inside it and in the surrounding space. The positive ions move in the direction of the lines of electric force and the negative ones—in the opposite direction with a velocity proportional to the density of these lines, i.e. to the field strength, constituting an electric current which tends to destroy the electrical charges of opposite sign arising on the upper and lower edges of the cloud, that is to depolarize the latter.

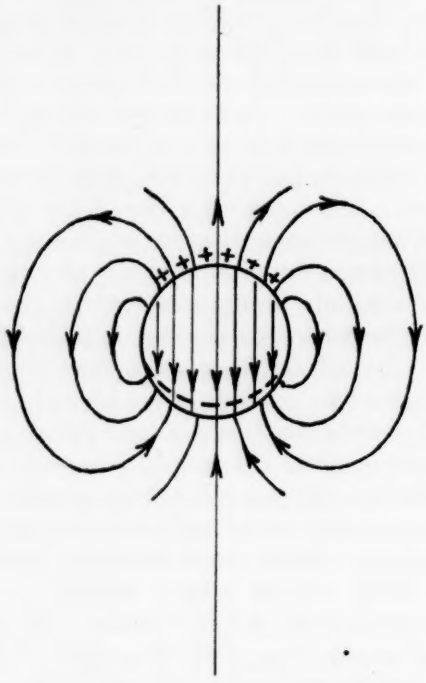

FIG. 2.

9. Electrical Field of the Cloud, Resulting from the Compensation between the Polarizing Effect of Gravity and the Depolarizing Effect of the Electrical Currents.

This depolarizing current, gradually increasing with increase of the polarization of the cloud, finally leads to a steady value of the polarization, which remains constant so long as the drops, constituting the cloud, keep sinking with respect to the air (i.e. so long as the cloud continues to exist, its downward motion being balanced by an upward flow of the air). The steady value of the polarization, corresponding to a balance between the action of gravity on the drops and that of the electrical forces acting both on these drops (upwards) and on the ions, is determined by the condition that the total current density inside the

cloud should vanish. This total density is equal to a sum of the convection current, carried by the negatively charged drops, and of the conduction current due to the electrical conductivity of the ionized air. The former part of the current density is equal to Nqv, where v is the velocity with which the water drops are sinking with respect to air, q—the (average) charge of a drop, and N—the number of drops per unit volume, while the latter part, according to Ohm's law, is equal to λE, $\lambda = \lambda_+ + \lambda_-$ being the specific conductivity of air and E the strength of the electrical field inside the cloud. The magnitude of this field strength, specifying the steady value of the cloud's polarization, is thus given by the formula

$$E = \frac{Nqv}{\lambda}.$$

The water content of a cloud amounts, as a rule, to 10^{-6} g./cm.^{3}. With a radius $r = 10^{-3}$ cm. of a drop this corresponds to $N = 200$ 1/cm.^{3} Putting $q = rV = 10^{-6}$ E.S.U. (that is, $V = 10^{-3}$ E.S.U. $\cong 0.3$ volt) and $\lambda = 2.10^{-4}$ sec.$^{-1}$ (electrical conductivity of air near the surface of the ground), and identifying v with the velocity 0.3 cm./sec. with which drops of the above size would fall under the influence of gravity alone, we obtain $E = 1$ E.S.U., that is about 100 volts/cm.[6]

This theoretical value of E is by an order of magnitude smaller than that which would be necessary in order to balance the weight of the drops by the electrical force pulling them upwards. Hence it follows that this force, under normal conditions, can be neglected, as has actually been done above.[7] Our theoretical value of E is in fair agreement with the experimental values of the field strength in the clouds, recently obtained by Simpson and his co-workers (with the help of a special instrument, called "altielectrograph," lifted by means of pilot balloons). It should be especially emphasized that our theory explains the seemingly surprising fact, discovered by Simpson, that the *average* value of the electric field strength is about the same both in ordinary clouds and in thunder clouds. We shall see later on that very high field strengths, which cause lightning discharges, are localized in relatively small regions of thunder clouds, and shall explain the origin of such localized abnormally high fields.

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[6] Putting $q = rV$ and $v = mg/6\pi\eta r$, according to Stokes' law (η—viscosity of the air $= 10^{-3}$), one can re-write the preceding formula as follows: $E = NmgV/6\pi\eta\lambda$. The product Nmg is the water content of air per cc. Hence it follows that for a given water content of the cloud, the field strength E is independent both of its size and of the size of the drops constituting it.

[7] In order to account for the action of the electrical forces, the weight of the drops mg must be decreased by qE, which corresponds to a decrease of the velocity of their sinking by a fraction qE/mg. With $q = 10^{-6}$, $m = 4.10^{-9}$ and $g = 10^3$, this fraction is approximately equal to 1/12, so that the correction due to the action of the field can be neglected.

10. Distortion of the Field, Generated by a Polarized Cloud under the Influence of the Ground.

Whereas inside the cloud the electrical field has a constant strength and direction (upwards), its direction in the outward space varies as shown in Fig. 4, while its strength decreases inversely as the cube of the distance from the centre (for a spherically shaped cloud). The strength of the field on the equator is the same inside and outside the cloud, while near the poles it is twice larger outside than inside it (the direction of the field being in this case opposite).

The picture of the electric field, shown in Fig. 4, refers to the case of a spherical cloud situated in an unlimited homogeneous medium. In reality the atmosphere is limited below by the surface of the ground, which, compared with the air, behaves as a practically ideal conductor (its electrical conductivity exceeding that of air by a factor of the order of 10^{14}).

If the surface of the ground is treated as plane, its influence on the electric field created by a point charge e, situated at a certain point P (above the ground), is equivalent to a point charge $-e$, concentrated at the symmetrical point Q under the ground. The resulting electrical field, due to the charge $+e$ and its "electrical image" $-e$, is represented by a system of lines of force, diverging from the point P and ending vertically on the surface of the ground. The negative charge $-e$ is thus actually distributed over the surface of the ground, and serves to compensate the field of the charge $+e$ below it.

The preceding method can easily be generalized to the case of an arbitrary system of electrical charges, distributed above the surface of the ground. Thus, for example, the influence of the ground on the electrical field produced by a polarized spherically shaped cloud is equivalent to the influence of the electrical image of the cloud, i.e. of a cloud of the same shape and size, situated symmetrically below the surface of the ground and polarized in the same direction (so that the lower negative edge of the original corresponds to the upper positive edge of the image, and the upper positive edge of the former to the lower negative edge of the latter). The resulting electrical field, shown in Fig. 3, is represented by a system of lines of force, which are directed in a relatively narrow region below the cloud from the ground towards the lower edge of the cloud, while in the remaining space they are directed from the upper edge of the cloud towards the ground. The boundary line between the two regions on the surface of the ground is a circle with a radius somewhat larger than that of the cloud (in the ratio $\sqrt{2}$ to 1). Within this circle the surface of the ground is positively charged, whereas outside it it is charged negatively, i.e. in the same way as is actually found in cloudless regions (where the electric field is known to have a downward direction).

The charges of the two regions must be numerically equal (so long as the charges of the upper and lower edges of the cloud are equal), so

that the resulting total charge of the earth's surface turns out to be equal to zero.

Under such conditions the average strength of the electric field near the surface of the ground in cloudless regions of the globe should be smaller than its average value in clouded regions in the inverse ratio of the areas occupied by these regions. A statistical study of the distribution of the clouds over the earth seems to indicate that this ratio lies close to 1/100. Since, according to the theory, sketched in the

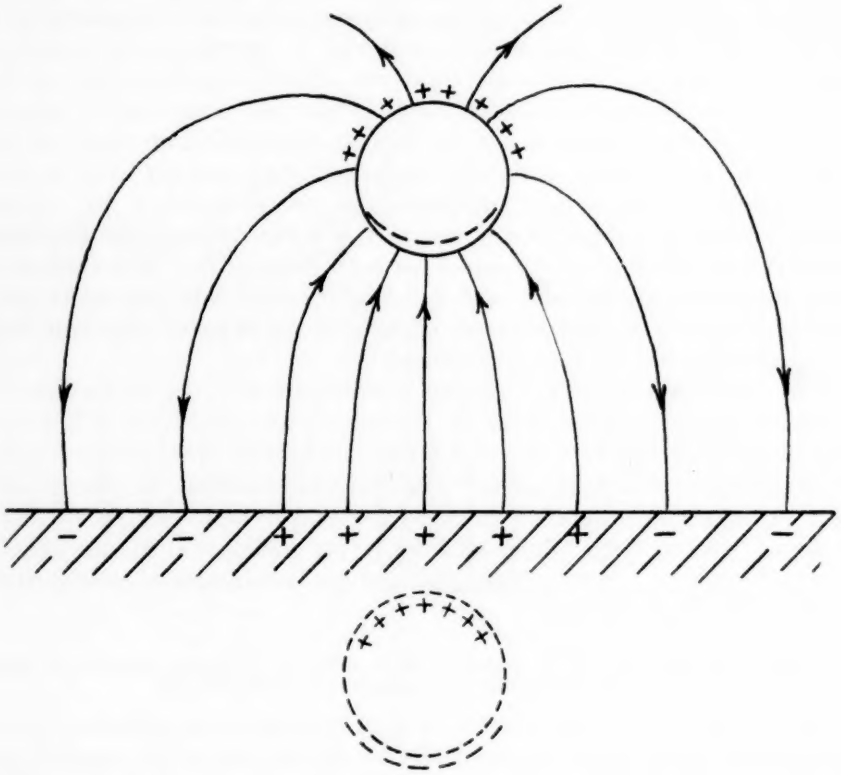

FIG. 3.

preceding section, the electrical field strength inside the clouds—and approximately in the space lying just below them—is of the order of 100 volts/cm., there follows that the strength of the electric field near the ground in cloudless regions must amount, on the average, to 1 volt/cm.—which is in agreement with the experimental facts.

11. The Closing of Atmospheric Electric Currents through the Ground.

If the atmospheric electric field, due to the polarization of the clouds, was not perturbed by any other factors, except the influence of the

ground, the strength of this field near the surface of the ground should decrease with increase of the distance from the centre of a cloud in a desert country inversely as the cube of the distance from this centre. In reality, such a decrease is *not* observed, and at large distances from clouded regions the electrical field near the ground preserves an approximately constant strength, which means that the surface of the ground in cloudless regions is charged with negative electricity distributed over it in a rather uniform way.

If the electrical conductivity of air had the same value throughout the whole atmosphere the deviation of the structure of the atmospheric electric field from the simple picture of Fig. 6, corresponding to a vanishing conductivity of the air, and an infinite conductivity of the ground, would reduce to the following. Near the surface of the ground the electric lines of force would be slightly inclined at an angle of the order of the ratio between the specific conductivity of the air and that of the ground; the vertical component of the field below and of the surface of the ground would stand in the same ratio, while the horizontal components would be equal on both sides of it. The extremely weak field inside the ground multiplied by its relatively extremely large electric conductivity, would generate electric currents of the same density as those flowing above the ground.

The density of electrical charge distributed over the surface of the ground is actually determined by the continuity condition of the vertical component of the currents flowing upwards in clouded regions and downwards in cloudless ones. The currents flowing in the ground should close up the circuits formed by the atmospheric electric currents, as shown by the dotted lines in Fig. 4 (the inclination of the lines of electric force—or current—near the surface is immensely exaggerated, for the sake of clearness).

12. Distortion of the Atmospheric Electric Field due to the Increase of the Conductivity of the Air with Altitude and Positive Electrification of the Atmosphere in Cloudless Regions.

If the distortion of the electric field of the clouds were limited to the influence of the different conductivity of the air and of the ground, the structure of this field would practically be the same as in the simplified case considered before (vanishing conductivity of the air, infinite conductivity of the ground). In reality, however, as has already been mentioned before, the conductivity of the air is different at different points. In the first place this conductivity increases with altitude, owing to the increase of the intensity of the ionizing (cosmic) rays. Under such conditions the steady state of the lower atmosphere, where the electric field, in the absence of clouds, is directed vertically downwards, can be maintained if the strength of this field varies inversely as the specific conductivity, so that the density of the air-earth current should be constant (i.e. independent of the altitude).

This relation between specific conductivity and field strength has been established experimentally. Now, the decrease of the field strength with increase of altitude (for moderate altitudes, in cloudless regions) implies a *positive* electrification of the air—so long as the atmospheric field is directed vertically downwards. In those regions where it has an upward direction (under a cloud, for example) its decrease must be associated with a *negative* charge of the air.

In order to explain the fact that in cloudless regions the atmospheric electric field has practically everywhere (except, probably the upper troposphere) a downward direction, the negative charge, compensating

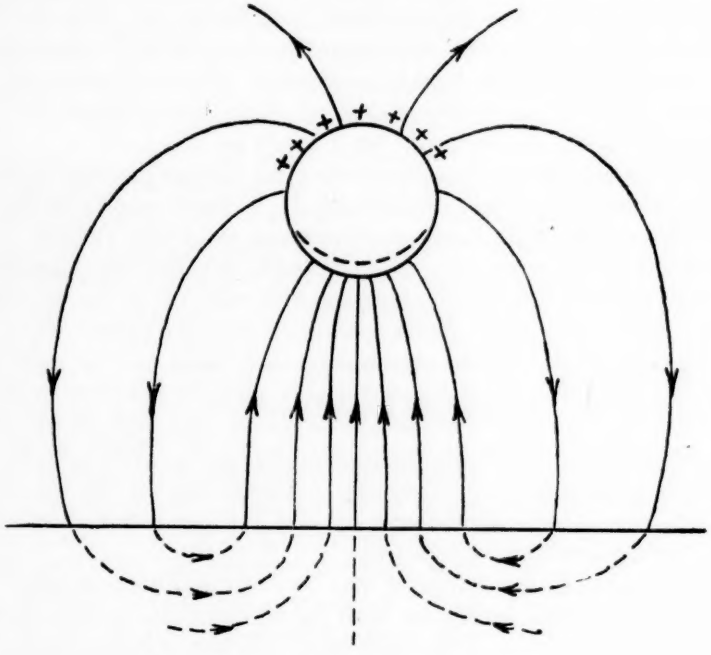

FIG. 4.

the positive electrification of the air, must be assumed to be concentrated on the surface of the ground in these very regions, and not the clouded ones. The negative charge of this surface must, accordingly, consist of two parts, one of them (with a variable density) compensating the positive charge of the upper edge of the surrounding clouds (as shown in Fig. 3), while the other (with a practically constant density) corresponds to the positive volume charge of the air.

The origin of this additional negative charge of the earth's surface is connected with the variation of the effective conductivity of the air in horizontal directions, namely with the fact that it is *larger under the clouds than in cloudless regions.*

13. Distortion of the Atmospheric Electric Field Owing to an Increased Effective Conductivity of Air under the Clouds and the Resulting Negative Charge of the Earth's Surface.

As has been mentioned before, the electric field near the surface of the ground in clouded regions is, on the average, about 100 times larger than in cloudless ones. In the case of thunder clouds the electric field reaches, locally, still higher strengths, which are sufficient for starting lightning discharges. Similar discharges, not of the spark but of the "glow"-type, arise in the presence of high electric fields at the tips of pointed objects connected with the ground—such as grass, twigs and leaves or needles of trees, vertical rods and pikes, masts and so on. The average electric field surrounding such objects is amplified near their pointed tips by several orders of magnitude, reaching the breakdown value for air. Under such conditions "corona" discharges are started, i.e. local glow discharges associated with luminescence of the surrounding air. This luminescence is sometimes observed, during lightning storms, in the form of "St. Elmo's fire."

In both cases—of the spark and the glow discharge—we have to do, essentially, with the same effect, namely a *local amplification* of the electric field, connected with an amplification of the electric current. The latter effect depends on the former not only directly (according to Ohm's law) but also indirectly, through the increase of the specific conductivity of the air in high fields (owing to ionization by collision).

We thus see, that when the *average* intensity of the atmospheric electric field is sufficiently increased—as is actually the case under thick clouds, like the thunder clouds—the average strength of the electric current due to this field, with account to its local amplification and the corresponding increase of the specific conductivity of the air, must be larger than that which corresponds to the average strength of the field and the normal conductivity of air.

In other words, the ratio of the average current density to the average field strength increases with increase of the latter—beyond a certain threshold value. This ratio will be denoted in the sequel as the "effective" conductivity of the air.

If the conductivity of air was everywhere the same (as was supposed before), the positive charge of the surface of the ground under a cloud would be exactly compensated by the negative charge of the surrounding surface, just as in the case of the absence of any currents (§ 10). In fact, the surface density of electrical charge at a certain point of the earth's surface, multiplied by a universal numerical coefficient (4π), is equal to the strength of the electric field at this point (more exactly to the change of its vertical component in a transition from air to ground).

On the other hand, the density of the electric current is likewise proportional to the field strength, the proportionality coefficient being in this case the specific conductivity. Under the condition of a steady electrical state of the earth—ground and atmosphere—the total strength

of the current, flowing under a cloud from the ground to the lower edge of the cloud, must be equal to the total strength of the current, flowing from the upper edge of the cloud to the remaining part of the surface of the ground. This relation remains valid for any number and distribution of clouds.

Now, what must occur when the (effective) conductivity of the air under the clouds is abruptly increased? It is clear, that if the field strengths over the clouded and cloudless regions are kept constant, the total strength of the electric currents flowing upwards in the clouded regions will become larger than that of the currents flowing downward in the cloudless ones. The balance between the two currents will thus be destroyed, and the average electric field in the clouded region will gradually subside, until the balance of the electric currents is re-established. As a result, however, the balance between the corresponding electric surface charged will be destroyed, the positive charge of the earth's surface under the cloud subsiding in the same ratio as the corresponding field (while the negative charge of the remaining area is slightly increased). The surface of the globe, as a whole, must thus acquire a net electric charge of a negative sign, distributed uniformly over its surface and superposed on local charges of both signs, which are due to the induction effect of the clouds.

14. General Scheme of the Earth's Steady Electrical Circulation.

Summing up the preceding considerations, we can state the results obtained as follows.

The clouds with their negatively charged water droplets or ice-crystals, sinking with respect to air under the influence of gravity, represent electro-gravitational generators, giving rise to the earth's electric field. These generators create in the lower layers of the atmosphere a system of electrical currents, which are closed up through the upper layers of the ground and which, under the conditions of steady state, determine the actual (distorted) distribution of electrical charges over the surface of the ground and in the atmosphere—in qualitative, and to a certain extent quantitative agreement with the empirical data. This agreement refers, in the first place, to the *direction* of the atmospheric electric field in the absence of clouds and under the clouds, and also to the absolute magnitude of the field in the corresponding regions.

This theory of the phenomena of atmospheric electricity could be designated as "circulational" or "dynamic," since it is based on the consideration of the steady current-system generated by the clouds. This circumstance distinguishes it from various theories, proposed thus far, which can be characterized as *statical*, for they all treated the atmospheric electric field as due to the negative charge of the earth's surface and of the positive charge of the air. According to our theory,

the primary sources of the atmospheric electric field are the electric charges of opposite sign arising on the upper and lower edges of the clouds, owing to the action of gravity on the particles constituting them, whereas the additional negative charges of the surface of the ground and of the lower atmosphere *in cloudless regions* are rather a *consequence* of the electric currents generated by the clouds, than the *cause* of the corresponding modification of their electric field.

From this point of view the main difficulty of the previous theories of atmospheric electricity—the problem of "the maintenance of the earth's negative charge"—turns out to be non-existent.

15. Rôle of Lightning and Glow-Discharges in the Electrical Circulation of the Atmosphere.

A number of scientists, and especially the well known British physicist C. T. R. Wilson, attempted to explain the maintenance of the negative charge of the cloud-free part of the earth's surface, in spite of the existence of the discharging air-earth current, with the help of the conception of a "counter-current," constituted by lightning discharges, and also glow discharges on those parts of the earth's surface which are covered by thunder clouds.

We now know that the electric field under such clouds actually has that "negative" (i.e. upward) direction, which is required by Wilson's hypothesis. The reason for this remained, however, unclear thus far.

From our point of view it becomes quite obvious. Moreover the notions of "discharging current" and "charging counter-current" become devoid of physical meaning, the "counter-current" being simply a part of the total depolarization current system, generated by the clouds and closing up through the ground. The spark and glow discharges (i.e. lightning strokes and corona discharges from point-like terrestrial objects), which are observed in the case of thunder clouds, constitute but a certain *fraction* of this discharging current in those regions where it is erroneously treated as a charging one. The true charging or, rather, polarizing-current is the *convection current*, formed by the sinking of the negatively charged cloud particles with respect to the air.[8]

The statistic of the thunder storms shows that the number of such storms existing simultaneously in various parts of the globe lies near 2,000. During each second the ground is struck, on the average, by 100 lightnings, each of which carries about 20–30 coulombs of negative electricity. The strength of the "counter-current," constituted by all these lightning strokes, amounts to 1,500 amperes. On the other hand, the total strength of the "discharging" current flowing from the air to the ground in cloudless regions is approximately equal to 1,700 amperes. The balance of electrical currents can thus practically be fully ensured

[8] It should be reminded once more, that the air can rise with respect to the surface of the ground, so that the cloud remains fixed in space.

by lightning discharges alone.—It must be noted, however, that the preceding estimates (due to Wilson) are extremely crude. According to other—better founded—calculations, the main part of the current under thunder clouds corresponds not to lightning, but to corona discharges from point-like objects. This question, as well as the question of the rôle played by ordinary conduction currents, both under thunder clouds and other clouds, not connected with lightning discharges, requires further study.

CHAPTER IV. LIGHTNING.

16. Glow Discharges and Short Electric Sparks.

The problem of lightning is thus reduced to the question as to the conditions under which the atmospheric electric current acquires the characteristics of a spark discharge or of a glow discharge.

The glowing or "corona" discharges are due to a local amplification of the electric field by grounded objects with pointed ends or, more generally, with a large positive curvature of the surface (thin wires, for example). The amplification coefficient can be so high that with an average field of, say, 100 volts/cm. over the plane surface of the ground the maximum intensity—near the point-like end of a grounded rod—can reach a value of the order of 30,000 volts/cm., which is necessary for the electric breakdown of air at normal pressure. The mechanism of this breakdown consists, as is well known, in the ionization of the neutral molecules by collisions with electrons, released by previous collisions or ultimately by the ionizing (cosmic) radiation, and accelerated to high velocities by the electric field acting on them.

The corona discharge is localized in the neighborhood of the highly curved portion of the surface, where the electric field is abnormally intense. It should be mentioned that a fraction of the electrons or ions, arising by the mechanism of secondary ionization in the corona, are drawn by the electric field to a relatively large distance outside it, causing an enhanced conductivity of the surrounding air.

With further increase of the average electric field, and, along with it, of the amplified field, which should be established near the tip of a rod-like object in the absence of breakdown, the character of the discharge begins to alter. In fact, the field strength cannot exceed the breakdown value (30,000 v./cm.) owing to the increased electrical conductivity of the air, which is reached under breakdown conditions. The highly ionized layer of air constituting the "corona" of the glow discharge, behaves practically in the same way as the conductor on which this corona arises; in other words, the glowing corona can be treated as a continuation of the rod-like conductor in the longitudinal direction. The effective tip of this conductor is thus constituted not by its geometrical pointed end, but by the outward edge of its corona, where the strength of the electric field falls below the breakdown value,

If, therefore, the average value of the field strength over the ground (or any other conducting body with protruding points) is increased, the corona of the glow discharge over the tip of a rod-like object is gradually extended. This is accompanied by a subsidence of the field strength inside the conductor, formed by the corona, and an increase of it on the external side of its tip exactly in the same way, as if the latter was the tip of a solid conductor.

In that case when the electric field is due to a potential difference between two plane electrodes with small protuberances, the "coronal continuation" of one of them finally reaches the second electrode. The two electrodes are thus connected with each other by a narrow channel of highly ionized air. This short-circuiting channel constitutes what is called an *electric spark*. An electric spark can thus be considered as the result of the extension of the corona of a glow discharge starting on a protuberance of one of the electrodes to the other electrode.

17. Long Electric Sparks and Lightning.

The preceding picture is, in principle, independent of the distance between the electrodes, i.e. of the length of the spark gap. So long as this gap is not too wide, the condition for sparking is reduced to a definite breakdown value of the field strength, namely 30,000 v./cm. in the case of air under normal temperature and pressure. For the breakdown of a spark gap 1 m. long, for example, a potential difference of 3 million volts is required. This law cannot, however, be extrapolated to arbitrary large distances between the electrodes. Thus, for example, from this point of view, a lightning stroke between the ground and the lower edge of a cloud at a height of 2 km. would be possible if the potential difference between them exceeded 7 billion volts, whereas, according to Simpson's measurements, the *average* field strength in a thunder cloud and in the underlying space is of the order 100 v./cm., which corresponds to a potential difference of the order of 70 million volts between the cloud and the ground.

The origin of a very long spark in the presence of a relatively weak (average) field is explained, just as the origin of glow discharges by a *local* amplification of the electric field till the breakdown value. This local amplification need not necessarily be due to the influence of conductors with a pointed tip. In the presence of such conductors—of metallic rods serving for lightning protection, for instance—spark discharges tend to be replaced by glow discharges, preventing the development of abnormally strong fields which are required for the origin of lightning. It seems that the latter is rendered possible by poor conductors, rather than by good (metallic) ones.

Without going into the details of this question, we shall mention the well established fact that lightning strokes arise not only on the ground but just as well in the clouds (striking either the ground, or from cloud

to cloud). In the former case the channel of heavily ionized air, constituting the lightning, grows rapidly upwards from the point of its origin on the surface of the ground; in the latter case it is extended from the cloud downwards (or sidewise). In both cases, however, the current flows in the same direction, namely from the positively charged surface of the ground towards the negatively charged (lower) portion of the cloud, which plays the rôle of the negative electrode. This current consists, as a rule, of several successive pulses with the same (upward) direction and a direction of the order of a few microseconds, the total duration of the discharge being of the order of 1/100 second. Each pulse weakens the field which has caused it; as a result of a series of such pulses, forming the lightning stroke, the electric field in the corresponding region subsides or even changes its sign, whereafter the channel of the current collapses and the lightning is extinguished.

18. The Connection of Lightning Discharges with the Process of Coalescence of Water Drops.

Both in the case of cloud to ground and cloud to cloud discharges the rôle of the cloud, just as that of the ground, consists in an abnormal amplification of the general field in relatively small regions till breakdown values. This amplification, in the case of the clouds, takes place, however, by a mechanism, entirely different from that described above (i.e. without the help of conductors with pointed tips), namely by *a coalescence of the water drops* (or ice-crystals) constituting the bulk of the cloud.

This process starts in the higher levels of the cloud by the coalescence of cloud particles of approximately the same size (which sink with the same velocity) owing to hydrodynamic forces. As soon, however, as larger particles are formed, which are falling down faster than the surrounding cloud particles of the standard size ($r = 10^{-3}$ cm.), they begin to grow at the cost of the latter, by catching them in their downward flight. The rate of this process rapidly increases as we pass from the higher levels of a cloud to the lower ones. As a result, in the lower levels of clouds with a very large thickness—such as thunder clouds—large drops (or crystals) appear, which can come down to the surface of the ground, without evaporating, in the form of rain (or snow).

If the electrical charges of the drops were preserved in the process of their coalescence, the charge of the larger drops, rapidly growing at the cost of the smaller ones, should increase directly as their volume, i.e. as *the cube of their radius*. On the other hand, in the case of equilibrium, which is characterized by a given electric potential of the drops, independent of their size, the charge of the drops should be proportional to the *first* power of their radius. The process of the growth of larger drops at the cost of the smaller ones thus leads to a distortion of the equilibrium or, rather, steady-state of the cloud. This distortion is partially removed by the scattering of the extra negative charge of the

larger drops in the surrounding air. The latter process lags, however, with respect to the former—the more so, the larger the drops and the rate of their sinking. Accordingly, in the middle or lower levels of a thick cloud an abnormally high concentration of negative charge, carried down by the larger drops, can arise. The polarization and consequently the electric field of the cloud in the corresponding region can therefore greatly exceed that which corresponds to the steady state; hence electric discharges both of the glow and of the spark type can start in these regions, extending to other clouds (or parts of the same cloud) and to the ground.

19. Nature of Thunder Clouds.

The preceding scheme (which has recently been worked out quantitatively by A. S. Shishkin and the writer), requires a number of corrections and complements.

In the first place it must be borne in mind that large water drops tend to acquire a positive charge owing to the induction effect of the atmospheric electric field (in connection with the larger contribution of the positive ions to the electric conductivity of the air (see § 7)). The increase of the negative electric potential of the growing drops due to their catching smaller ones, must therefore finally slow down and eventually be replaced by an increase of their positive charge. So long as the rate of this process varies from one portion of the cloud to another even at the same level (depending on the local thickness of the cloud, its water content and other circumstances), the abnormal increase of the strength of the electric field can be limited to separate regions of a relatively small extension, where the electric discharges are initiated.

Further, it seems to follow from the connection between the origin of lightning discharges and the formation of rain (or snow) that each rain cloud must be a source of lightning discharges, i.e. be a thunder cloud. Now, in reality, the converse relation only is observed—each thunder storm being, as a rule, connected with rain, while even heavy downpours are but rarely associated with electric storms, and this practically in the hot season only.

The latter circumstance is explained, according to our scheme, by the fact that the onset of very high local electric fields is possible in the case of clouds with a very large vertical thickness only, since the growing drops must sink over very large distances through the cloud—of the order of several kilometers—in order to acquire an excessively large negative charge.

As a matter of fact, thunder clouds are distinguished from ordinary rain clouds by their exceptionally large thickness which can exceed 6 km. This explains their characteristic dark appearance (which is partially due to the relatively high concentration of larger drops, scattering light much more intensely than smaller ones, for the same water

content of the cloud). The fact that sometimes lightning discharges are observed in the absence of rain is probably explained by the low relative humidity of the lower layers of the atmosphere, so that the rain drops are evaporated before they reach the ground.

The origin of thunder clouds is probably connected with a high temperature of the air near the surface of the ground, along with a large absolute humidity. Under such conditions, powerful ascending air currents arise, which are followed by the condensation of the water vapour at higher altitudes. Although the question of the mechanism of cloud formation is one of the main problems of meteorology, it remains thus far in a quite unsettled state. A satisfactory quantitative theory of electric storms can be developed only after this question is fully elucidated.

Mine Will Ship 20,000 Tons of Coal a Day.(*Blast Furnace & Steel Plant*, Vol. 34, No. 10.)—The world's largest coal mine, the Robena, now is in production at its own water front tipple, the H. C. Frick Coke Co., subsidiary of United States Steel, has announced.

The new dock at Robena Mine, located 80 river miles south of Pittsburgh, will feed thousands of tons of bituminous coal daily into barges destined for steel plants down the Monongahela river.

Eventually this mine is scheduled to ship 20,000 tons of coal a day. Mines producing 4,000 to 10,000 tons a day are normally considered big.

The entire output at the mine mouth is handled by an intricate system of heavy rubber conveyor belts, some six feet wide. Coal is transported from the working "face" in modern steel cars, pulled by electric locomotives. The mine has two rotary dumps into which the coal is fed at the base of a 300-foot slope leading to the outside. Each rotary can handle 10 carloads at a time. In them, high and lower sulfur coals are blended in the first of several steps taken to insure uniformity of the coal for producing metallurgical coke.

The rotaries empty onto a five-foot belt which conveys the product to the surface, where it is elevated to the top of a tall building housing shaker screens, picking tables, and crushers. Here, slate and other coarse refuse are removed and the coal is crushed to size three inches or smaller. An automatic sampler takes off test samples periodically.

The coal is then delivered by another belt system to the top of an 18,000-ton blending building, where a shuttle belt assembly distributes it into deep, concrete blending bins. At present, the coal is moved from the bottom of these bins to the top of the river tipple into loading bins from which chutes deposit it into river barges. The tipple has seven chutes which, when all are in use, can fill a barge with 900 tons of coal in about six minutes.

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AN APPLICATION OF THE THEORY OF THE COMPLEX VARIABLE TO THE MAPPING OF SOME POTENTIAL FUNCTIONS.

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ABSTRACT.

Using the theory of the logarithmic potential and the methods of intersecting circles and radii, plots are obtained for the lines of flow and equipotentials for eight cases of two sources, two sinks, or a source and a sink of varying strengths. A detailed analysis of the equilibrium equipotentials is included. The problem of three unit sources placed at the vertices of an equilateral triangle is investigated; and the general expression of the equilibrium equipotential for n similar unit sources (or sinks) placed at the vertices of a regular n-sided polygon is developed.

INTRODUCTION.

In his treatise, "Electricity and Magnetism," James Clerk Maxwell was concerned with drawing "several diagrams of systems of equipotential surfaces and lines of induction, so that the student may make himself familiar with the forms of the lines." [1] Unfortunately, Maxwell included only eight such figures in Volume I; and despite the many years which have elapsed since 1904 (the date the third edition was published), very little additional material of this nature has appeared in the literature. In this paper, some classically familiar cases and some hitherto unpublished illustrations are plotted and analyzed.

Here we consider given: a source and a sink, two sources, or two sinks of the same or different strengths. We plot the lines of flow and the equipotentials and investigate the associated equations using the general theory of conjugate functions and particular logarithmic functions. The method of intersecting circles outlined by Maxwell for obtaining the plots in ordinary potential is also employed but as it applies to the theory of logarithmic potential.

As an additional interesting application, the case of three sources (or sinks) of equal strengths is also developed in detail.

Laplace's equation in two dimensions is $\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = 0$. If w is an analytic function of a complex variable, say, $w = u(x, y) + iv(x, y)$, then Laplace's equation gives rise to $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ and $\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0$.

The harmonic conjugates or solutions of these equations represent two mutually orthogonal, simply infinite families of curves $u = c$ and $v = C$, where c and C are arbitrary constants.

[1] James Clerk Maxwell, "Electricity and Magnetism" (1904), I, 177.

As an example, using the particular function $w = \log z$ and writing $w = u + iv$ and $z = \rho e^{i\theta}$, $w = \log \rho + i\theta$ and consequently $u = \log \sqrt{x^2 + y^2}$, $v = \tan^{-1} y/x$.

Since the Cauchy-Reimann differential equations are satisfied at all points except $x = 0$ and $y = 0$, this logarithmic function is holomorphic in the finite region (except for $z = 0$). Because the logarithmic function is analytic, the mapping of the z-plane upon the w-plane gives a conformal representation.

Plotting $u = c$ and $v = C$, two families of perpendicular straight lines in the w-plane (Fig. 1) are obtained. In the z-plane, the plot $u = c$ or $x^2 + y^2 = e^{2c}$ represents a family of concentric circles with the center at the origin; and $v = C$ yields $y = x \tan C$ which is a family of straight lines passing through the origin (Fig. 2).

Obviously, Fig. 2 may represent a source at the origin and a sink at an infinite distance. The radii in this case represent the lines of flow; and the concentric circles the equipotentials.

It is possible to extend the logarithmic function to express the general case of any number of sources and any number of sinks each having a given strength. Since sinks may be considered negative sources, they will occur in the denominator of the logarithmic function. Therefore, assuming a source at each of the points $\alpha_1, \alpha_2, \cdots, \alpha_n$ having strengths $l_1, l_2, \cdots, l_n$ respectively, and a sink at each of the points $\beta_1, \beta_2, \cdots, \beta_n$ having strengths $\lambda_1, \lambda_2, \cdots, \lambda_n$ respectively, the general expression is

$$w = \log \frac{(z - \alpha_1)^{l_1}(z - \alpha_2)^{l_2} \cdots (z - \alpha_n)^{l_n}}{(z - \beta_1)^{\lambda_1}(z - \beta_2)^{\lambda_2} \cdots (z - \beta_n)^{\lambda_n}},^{2}$$

where $w = u + iv$.

Here the conjugate functions, u and v, are considered to be functions of x and y only and the investigations are confined to the x-y plane.

METHODS OF INTERSECTING CIRCLES AND INTERSECTING RADII FOR TWO SOURCES, TWO SINKS, OR A SOURCE AND A SINK.

Maxwell outlined methods of intersecting circles for drawing equipotentials and intersecting radii for obtaining lines of flow when working with ordinary potential.[3] In this paper these methods are extended to apply to the logarithmic function.

It has already been shown that the case of a single source (or sink) yields a system of concentric circles and their radii as the equipotentials and lines of flow.

Now consider the case of a source and a sink of equal strengths, say, two electric charges. Taken independently at an infinite distance from each other, the patterns would each be identically Fig. 2. However,

[2] E. J. Townsend, "Functions of a Complex Variable" (1915), 141.

[3] Maxwell, *op. cit.*, 183.

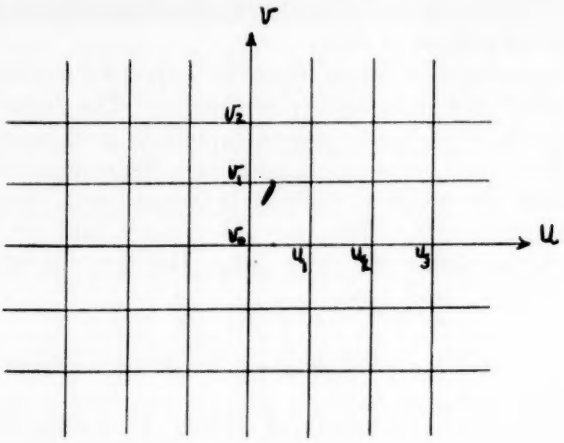

FIG. 1. The maps of $u = c$ and $v = C$ upon the w-plane.

if the charges are moved so that they are arbitrarily close to each other, the circles of these separate units will tend to overlap. If a unit plus charge were to be placed at a point of intersection, it would tend to move in the resultant direction of the forces acting on it. Thus the points of intersections of the families of the original circles would trace

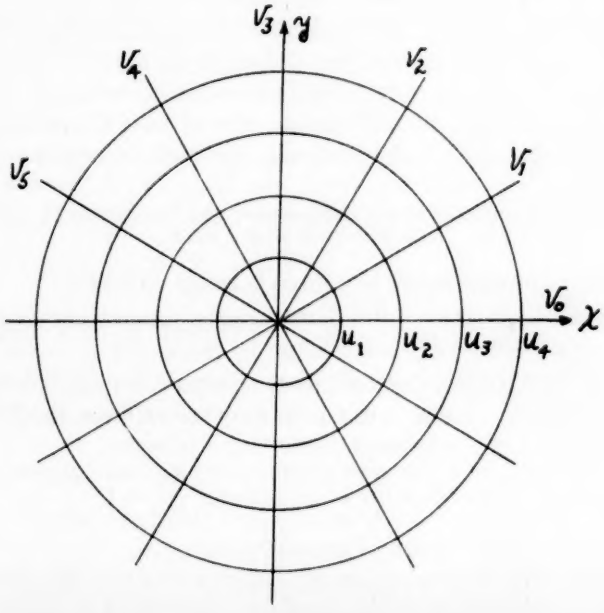

FIG. 2. The maps of $u = c$ and $v = C$ upon the z-plane.

the resulting equipotentials. Similarly, the intersections of the radii would yield the new lines of flow.

As the fundamental problem, consider only two sources, two sinks, or a source and a sink of arbitrary strengths. The function will then be $w = \log (z - \alpha_1)^{L_1}(z - \alpha_2)^{L_2}$ where L_1 and L_2 may be either positive or negative. Let $|\alpha_1| = |\alpha_2| = 1$ and place the sources or sinks on the horizontal axis so the origin is midway between them, then $w = u + iv = \log (z - 1)^{L_1}(z + 1)^{L_2}$. Put $(z - 1) = \rho_1 e^{i\theta_1}$ and $(z + 1) = \rho_2 e^{i\theta_2}$, then $u = \log \rho_1^{L_1}\rho_2^{L_2}$ and $v = L_1\theta_1 + L_2\theta_2$. But if $u = c$ and $v = C$, then

$$\rho_1^{L_1}\rho_2^{L_2} = e^c = K, \quad K > 0 \tag{1}$$

$$L_1\theta_1 + L_2\theta_2 = C. \tag{2}$$

To plot any one of the family of curves (1), assign to K any arbitrary value and allow ρ_1 to take on any succession of values; then compute the corresponding values for ρ_2. Using $z = 1$ as center, draw circles of radii equal to ρ_1. Superimpose on these the family of circles corresponding to radii ρ_2 and center $z = -1$; the intersections of these circles give the points on the resultant equipotential. Repeat this process for various arbitrary K's to obtain the family of equipotentials represented by (1).

To plot any one of the family of curves (2), assign to C any arbitrary value and allow θ_1 to take on any succession of values from 0 to π, then compute the corresponding values for θ_2. From the point $z = 1$, draw straight lines making an angle θ_1 with the real axis. Superimpose on this the family of lines making the angle θ_2 with the horizontal axis and emanating from $z = -1$. The intersections of these lines give the points on the resultant line of force. To obtain the family of curves represented in (2), repeat this process for various arbitrary C's.

ANALYSIS ABOUT THE POINT OF EQUILIBRIUM FOR TWO SOURCES, TWO SINKS, OR A SOURCE AND A SINK.

At a point of equilibrium, the force is zero. Thus $F = -\text{grad } u = 0$, or $i\dfrac{\partial u}{\partial x} + j\dfrac{\partial u}{\partial y} = 0$, hence $\dfrac{\partial u}{\partial x} = \dfrac{\partial u}{\partial y} = 0$ and it is a singular point. Therefore at the point of equilibrium a single equipotential intersects itself. Conversely, where an equipotential crosses itself is an equilibrium point and the resultant force there is zero.

The solution of the system of equations $\dfrac{\partial u}{\partial x} = 0$, $\dfrac{\partial u}{\partial y} = 0$ determines the coordinates of the point of equilibrium.

Using the function $w = \log (z - 1)^{L_1}(z + 1)^{L_2}$, the point of equilibrium is at $y = 0$, $x = \dfrac{-(L_1 - L_2)}{L_1 + L_2}$. Hence the value of K^2 asso-

ciated with the equilibrium equipotential is

$$K^2 = \left(\frac{-2L_1}{L_1 + L_2}\right)^{2L_1}\left(\frac{2L_2}{L_1 + L_2}\right)^{2L_2}. \tag{3}$$

It is of interest to investigate the equations of the tangents to the equilibrium equipotential at the point of equilibrium. The equation of the equilibrium equipotential is

$$[(x-1)^2 + y^2]^{L_1}[(x+1)^2 + y^2]^{L_2} = \left(\frac{-2L_1}{L_1 + L_2}\right)^{2L_1}\left(\frac{2L_2}{L_1 + L_2}\right)^{2L_2}. \tag{4}$$

Differentiating and solving for y', y' equals $\frac{0}{0}$. Since the slope is indeterminate, the equilibrium point is a singular point. To determine the slope at this point, translate (4) so that the equilibrium point is at the origin of the new coordinates. That is, put $x = X - \left(\frac{L_1 - L_2}{L_1 + L_2}\right)$ and $y = Y$. Then (4) becomes

$$\{[X(L_1 + L_2) - 2L_1]^2 + (YL_1 + YL_2)^2\}^{L_1}\{[X(L_1 + L_2) + 2L_2]^2 + (YL_1 + YL_2)^2\}^{L_2} = (-2L_1)^{2L_1}(2L_2)^{2L_2}. \tag{5}$$

This equation represents the equilibrium equipotential translated so that the point of equilibrium falls on the origin of coordinates. In general, if such a polynomial lacks the constant term and the first degree term, its singular point (at the origin) is of the same order as the degree of the term of lowest degree appearing in the equation. In this case, the constant term and the first degree term, both, drop out and the lowest appearing degree is two, which indicates a double point. To calculate the slopes of the two tangents, put $Y = mX$ and equate the new coefficient of X^2 to zero and solve for m. This yields $m = \pm 1$. Therefore the equations of the tangents are $Y = \pm X$, or in the original coordinates $y = \pm\left[x + \frac{L_1 - L_2}{L_1 + L_2}\right]$.

Thus the equilibrium equipotential for two charges crosses itself with a 45° slope at the point of equilibrium; and since the values of m are real and distinct, the double point is a node and the branches of the curve must cross each other.

RESULTS OF SPECIAL CASES OF TWO SOURCES, TWO SINKS, OR A SOURCE AND A SINK.

The accompanying table encompasses a condensation of the important equations and constants associated with the investigations of certain special cases of the aforementioned problem. (The diagram numbers of the lines of force and the equipotentials for each case are given in the last column of the table.)

Case No.	The Problem	Equation of the Equipotentials	Equation of the Lines of Flow	Point of Equilibrium	K for Equilibrium Equipotential	Diagram No.
I	Two sources of equal unit strengths.	$x^4+y^4+2x^2y^2-2x^2+2y^2+1=K^2$	$\frac{2xy}{x^2-y^2-1}=\tan C$	0, 0	1	3
II	A source and a sink of equal unit strengths.	$\left[x-\frac{1+K^2}{1-K^2}\right]^2+y^2=\left[\left(\frac{1+K^2}{1-K^2}\right)^2-1\right]$	$x^2+\left(y-\frac{1}{\tan C}\right)^2=\csc^2 C$	infinity	—	4
III	Two sources of strengths one and two.	$[(x-1)^2+y^2]^2[(x+1)^2+y^2]=K^2$	$\frac{3x^2y-2xy-y-y^3}{x^3-x^2-x+1-3xy^2+y^2}=\tan C$	$-1/3, 0$	32/27	5
IV	A source of strength two and a sink of strength one.	$[(x-1)^2+y^2]^2=K^2[(x+1)^2+y^2]$	$\frac{x^2y+2xy-3y+y^3}{x^3-x^2-x+1+xy^2-3y^2}=\tan C$	$-3, 0$	8	6
V	Two sources of strengths one and three.	$[(x-1)^2+y^2]^3[(x+1)^2+y^2]=K^2$	$\frac{2y[y^2(1-2x)+(x-1)^2(1+2x)]}{x^4-2x^3+2x-1-6xy^2(x-1)+y^4}=\tan C$	$-1/2, 0$	27/16	7
VI	A source of strength three and a sink of strength one.	$[(x-1)^2+y^2]^3=K^2[(x+1)^2+y^2]$	$\frac{y[2x^3-6x+4-y^2(4-2x)]}{x^4-2x^3+2x-1+6y^2(1-x)-y^4}=\tan C$	$-2, 0$	27	8
VII	Two sources of strengths two and three.	$[(x-1)^2+y^2]^3[(x+1)^2+y^2]^2=K^2$	$\frac{y[y^3-2y^3(5x^2-2x-1)+(5x^4-4x^3-6x^2+4x+1)]}{y^4(5x-1)-2y^2(5x^3-3x^2-3x+1)+(x-1)(x^2-1)^2}=\tan C$	$-1/5, 0$	3456/3125	9
VIII	A source of strength three and a sink of strength two.	$[(x-1)^2+y^2]^3=K^2[(x+1)^2+y^2]^2$	$\frac{y[x^4+4x^3-6x^2-4x+5+y^2(2x^2+4x-10)+y^4]}{(x^4-2x^2+1)(x-1)+y^2(2x^3-6x^2-6x+10)+y^4(x-5)}=\tan C$	$-5, 0$	27/2	10

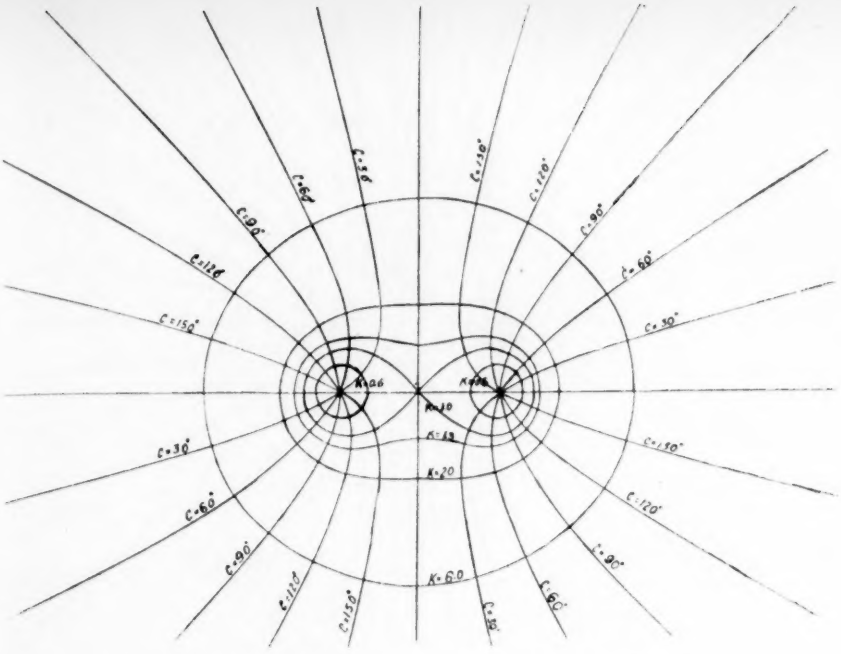

FIG. 3. The equipotentials and the lines of flow for two sources of equal unit strengths. (Case I.)

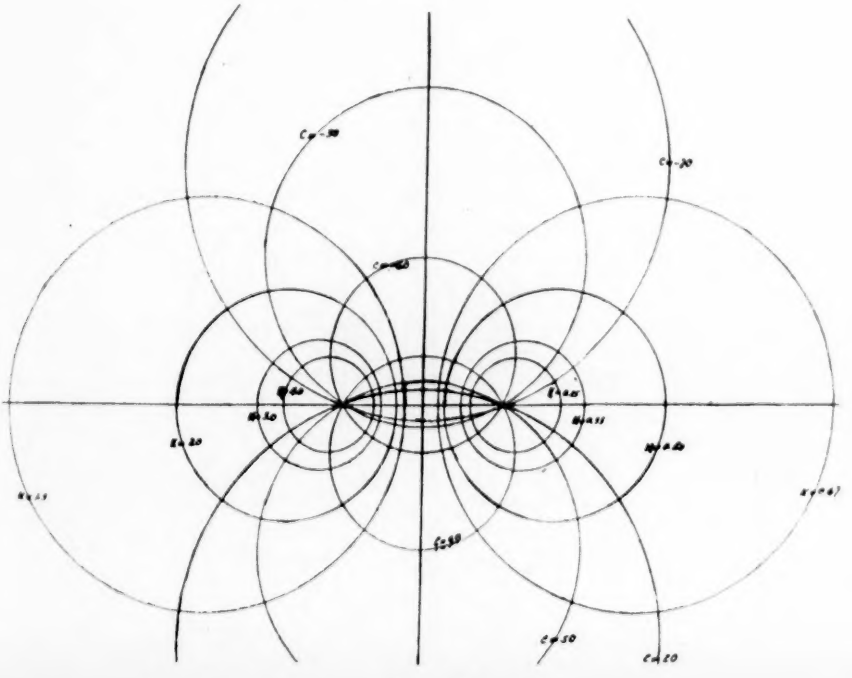

FIG. 4. The equipotentials and lines of flow for a source and a sink of equal unit strengths. (Case II.)

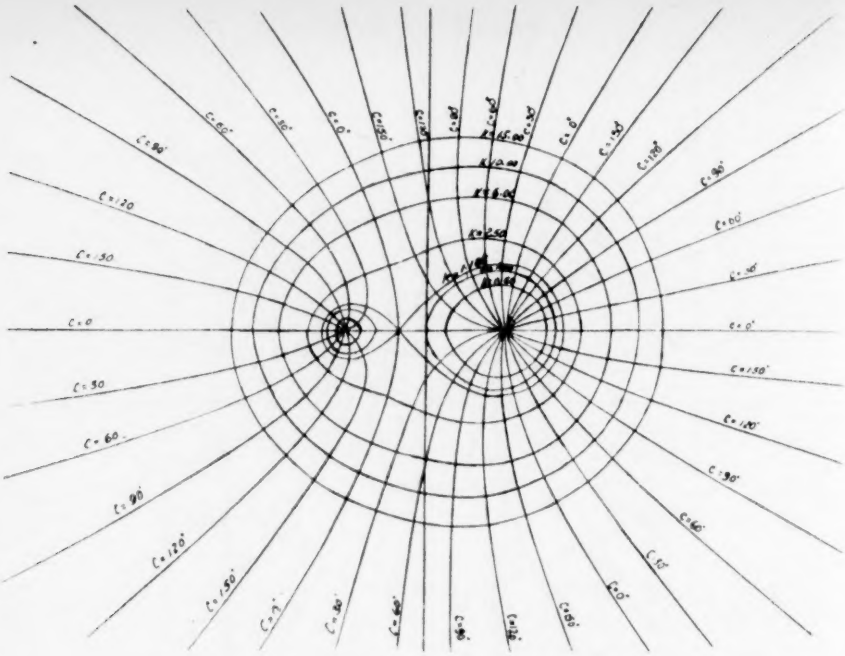

FIG. 5. The equipotentials and lines of flow for two sources of strengths one and two. (Case III.)

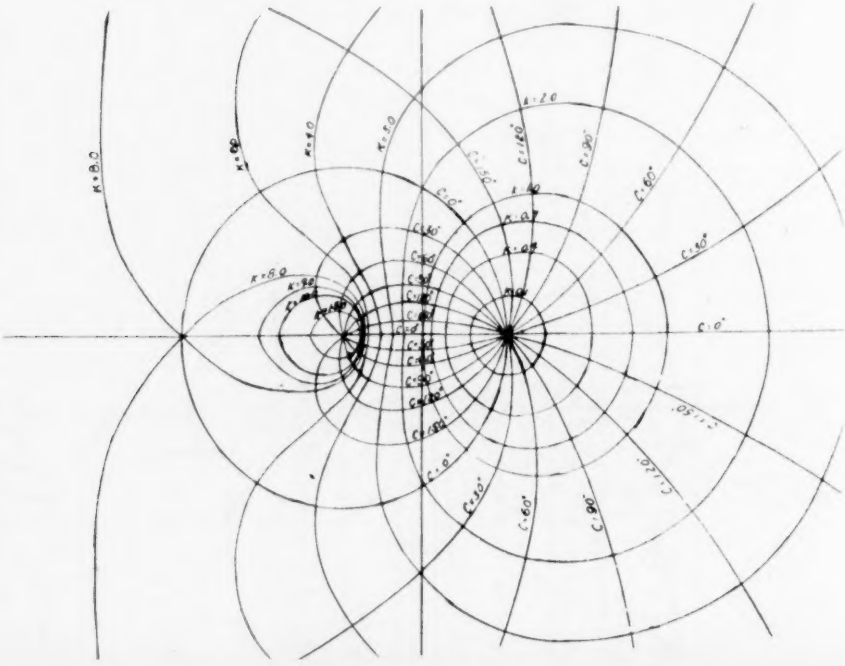

FIG. 6. The equipotentials and lines of flow for a source of strength two and a sink of strength one. (Case IV.)

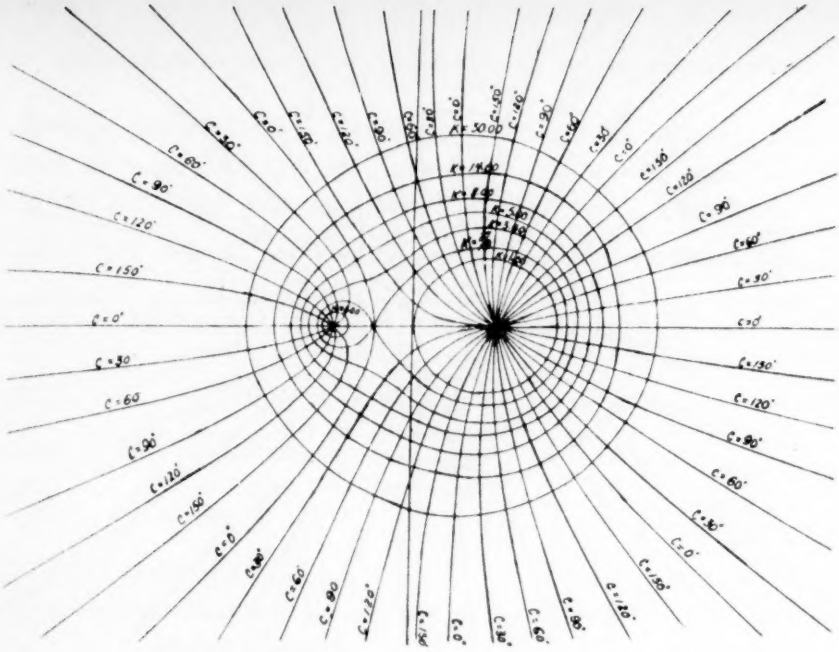

FIG. 7. The equipotentials and lines of flow for two sources of strengths one and three. (Case V.)

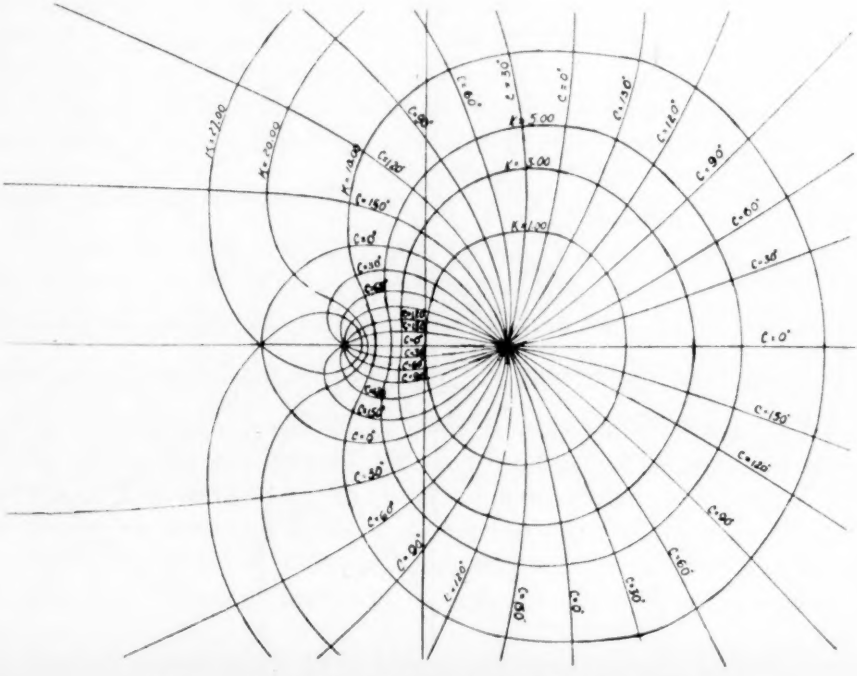

FIG. 8. The equipotentials and lines of flow for a source of strength three and a sink of strength one. (Case VI.)

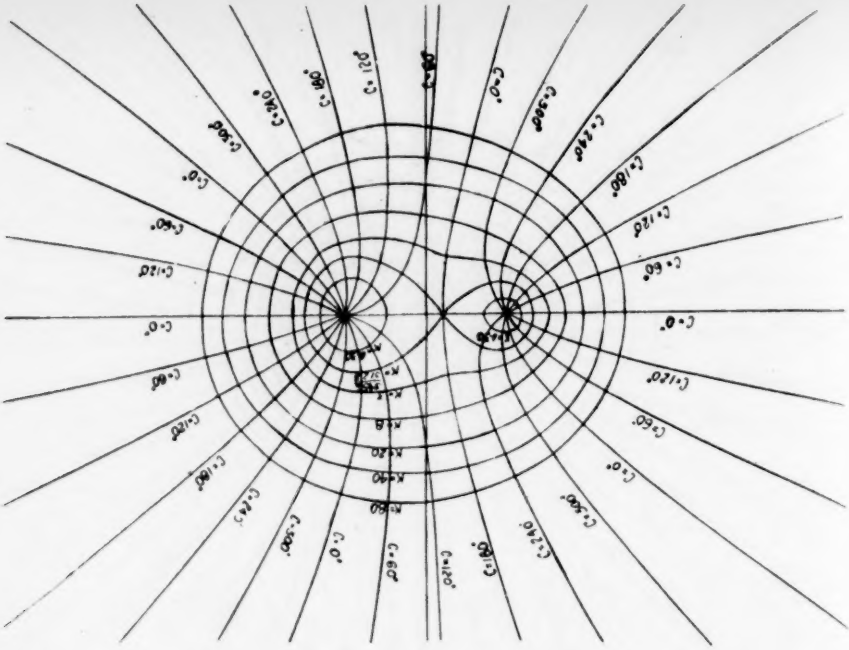

FIG. 9. The equipotentials and lines of flow for two sources of strengths two and three. (Case VII.)

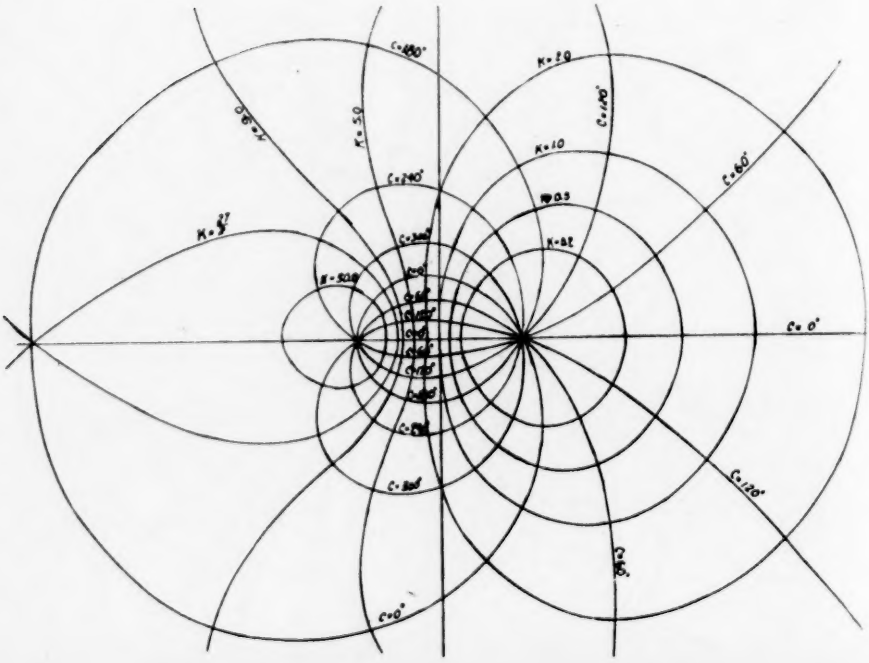

FIG. 10. The equipotentials and lines of flow for a source of strength three and a sink of strength two. (Case VIII.)

THE PROBLEM OF THREE EQUAL SOURCES PLACED AT THE VERTICES OF AN EQUILATERAL TRIANGLE.

As has already been outlined, the fundamental equation may be written $w = \log (z - 1)(z + 1)(z - \sqrt{3}i)$. Expressing the real and imaginary parts, $u = \log \rho_1\rho_2\rho_3$ and $v = \theta_1 + \theta_2 + \theta_3$. Hence $\rho_1\rho_2\rho_3 = K$ and $\theta_1 + \theta_2 + \theta_3 = C$. Solving $\frac{\partial u}{\partial x} = 0$ and $\frac{\partial u}{\partial y} = 0$ simultaneously, the point of equilibrium is at $\left(0, \frac{\sqrt{3}}{3}\right)$. Then K for the equilibrium equipotential is $\frac{8\sqrt{3}}{9}$. Hence the equation of the curve which represents the equilibrium equipotential is

$$[(x - 1)^2 + y^2][(x + 1)^2 + y^2][x^2 + (y - \sqrt{3})^2] = \left(\frac{8\sqrt{3}}{9}\right)^2.$$

Making the substitutions $x = r\cos\theta$, $y = r\sin\theta + \frac{\sqrt{3}}{3}$, this equation reduces to $r^3 = -\frac{16\sqrt{3}}{9}\sin 3\theta$. This is a sinusoidal spiral and it will be shown that, in general, for similar unit charges placed at the vertices of a regular n-sided polygon, the equilibrium equipotential is a sinusoidal spiral of the form $r^n = 2R^n \cos n\theta$.

Upon determining the slope of the equilibrium equipotential at the point of equilibrium, it is found that this singular point is a triple point and the tangents here are $y = \frac{\sqrt{3}}{3}$; $y = \sqrt{3}x + \frac{\sqrt{3}}{3}$; $y = -\sqrt{3}x + \frac{\sqrt{3}}{3}$.

To plot the equipotentials, assign to K and ρ_3 arbitrary values, then determine the pairs of factors representing $\rho_1\rho_2$. Draw circles of radii ρ_1 with $z = 1$ as center; those of radii ρ_2 with $z = -1$ as center; and those of radii ρ_3 with $z = \frac{\sqrt{3}}{3}$ as center. Superimpose the circles of ρ_1 and ρ_2; trace the curves given by these points of intersection. On this family of curves, superimpose the circles for ρ_3. The points of intersection of this set will yield the desired equipotential. Repeat this procedure for various K's to obtain a family of equipotentials.

Similarly, plot the mutual intersections of the three families of straight lines corresponding to θ_1, θ_2, θ_3, to obtain the family of lines of force.

The complete map of the superimposed plots of the lines of flow and the equipotentials is given in Fig. 11.

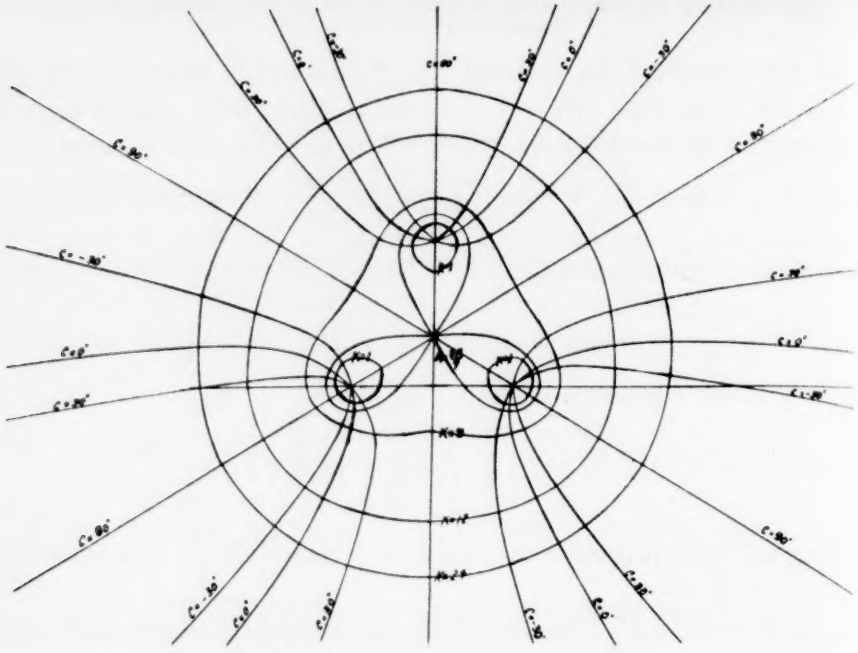

FIG. 11. The equipotentials and lines of flow for three equal sources placed at the vertices of an equilateral triangle.

THE GENERAL EQUILIBRIUM EQUIPOTENTIAL FOR *N* SIMILAR UNIT SOURCES PLACED AT THE VERTICES OF A REGULAR *N*-SIDED POLYGON.

To show that the equilibrium equipotential for similar unit sources placed at the vertices of a regular n-sided polygon is a sinusoidal spiral of the form $r^n = 2R^n \cos n\theta$, consider Fig. 12. First, let us show that the point of equilibrium is the geometric center of the polygon. Consider the vector equation $F = f_1 + f_2 + \cdots + f_n$ where f_n denotes the force exerted by each source, placed at each vertex, on the test unit placed at (0, 0). Rewriting this equation in vector notation using the i-j-rōt-vector method,* we have

$$F = f[i\underline{/-90} + i\underline{/-90+\alpha} + i\underline{/-90+2\alpha} + \cdots + i\underline{/-90+(n-1)\alpha}],$$

where

$$f = |f_1| = |f_2| = \cdots = |f_n| = \frac{1}{R} = \frac{1}{\frac{a}{2}\csc\frac{\pi}{n}},$$

* The operator $\underline{/\theta}$ was first extended to the i-j-rōt-vector method by Dr. Newton Gaines in the Physics Department of Texas Christian University in 1937 and has since been extensively used in the solution of problems involving vectors by his students.

$$F = f\{i[\cos(-90) + \cos(-90+\alpha) + \cos(-90+2\alpha) + \cdots + \cos<-90+(n-1)\alpha>] + j[\sin(-90) + \sin(-90+\alpha) + \sin(-90+2\alpha) + \cdots + \sin<-90+(n-1)\alpha>]\},*$$

$$F = f\left\{i\left[\frac{\sin\frac{n\alpha}{2}\sin(n-1)\frac{\alpha}{2}}{\sin\frac{\alpha}{2}}\right] - j\left[\frac{\cos(n-1)\frac{\alpha}{2}\sin n\frac{\alpha}{2}}{\sin\frac{\alpha}{2}}\right]\right\},$$

$$F = f\left\{i\left[\frac{\sin\frac{n\cdot 2\pi}{2n}\sin(n-1)\frac{2\pi}{2n}}{\sin\frac{2\pi}{2n}}\right] - j\left[\frac{\cos(n-1)\frac{2\pi}{2n}\sin\frac{n\cdot 2\pi}{2\cdot n}}{\sin\frac{2\pi}{2n}}\right]\right\},$$

$$F = 0.$$

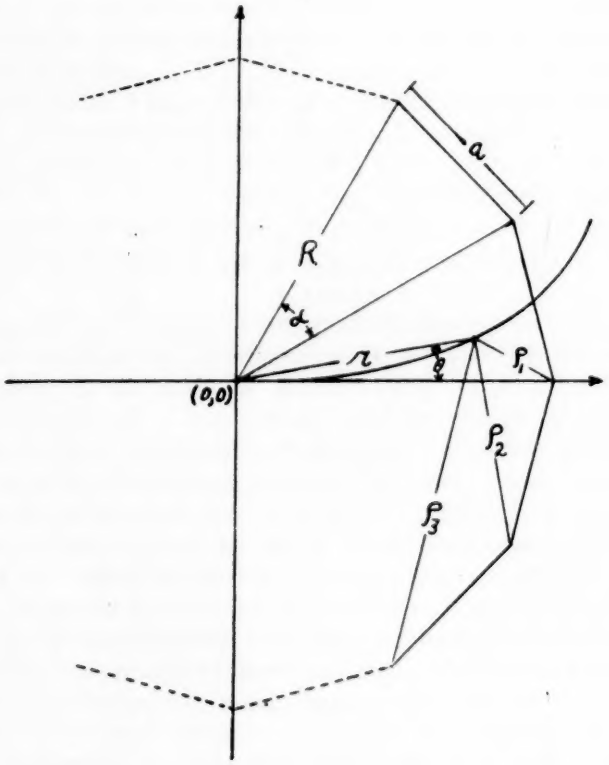

FIG. 12. Diagram used for deriving the sinusoidal spiral.

* The symbol < > here is a sign of grouping and should not be confused with the signs of inequality.

Since the intensity at (0, 0) is zero, that point must be a point of equilibrium.

The equation of the equipotentials is given by $\rho_1\rho_2\rho_3 \cdots \rho_n = K$. From Fig. 12, this equation may be written in the form

$$K^2 = \prod_{m=1}^{n} \{r^2 + R^2 - 2rR\cos[\theta + (m-1)\alpha]\}$$

where $\prod$ denotes the repeated product. Letting $m - 1 = k$, then

$$K^2 = \prod_{k=0}^{n-1} \left\{r^2 + R^2 - 2rR\cos\left[\theta + 2\pi\frac{k}{n}\right]\right\},$$

whence $K^2 = r^{2n} - 2r^nR^n\cos n\theta + R^{2n}$. Applying the boundary condition $r = 0$, $\theta = 0$, we have $K^2 = R^{2n}$. Hence the general equation of the equilibrium equipotential for this case is given by $r^n = 2R^n\cos n\theta$ which is the general sinusoidal-spiral equation.

DISCUSSION.

The patterns corresponding to two sources or two sinks bear resemblance to each other. Comparing Figs. 3, 5, 7, and 9, it is seen that the equilibrium equipotentials of Figs. 5, 7, and 9 seem like distorted lemniscates; of course, the equilibrium equipotential of Fig. 3 is a true lemniscate. The other curves follow a similar variation with respect to the Cassinian Ovals of Fig. 3.

For the cases of one source and one sink, the resemblance is also marked, except for the case of equal strengths which is an extreme and the point of equilibrium is at infinity.

Whereas the coordinates of the point of equilibrium for two charges depends only on the ratio of the strengths of the charges, the value of K^2 for the equilibrium equipotentials depends on the values of the strengths and not merely on their ratios.

Jeans,[4] using ordinary potential, investigated the problem of three equal charges placed at the vertices of an equilateral triangle. A comparison of Fig. 11 with Fig. 21 in Jeans shows that although the general configuration is similar, the three points of equilibrium have coalesced into a single point when logarithmic potential is used.

Much of this paper was written in partial fulfillment for a Degree of Master of Arts in the Department of Mathematics at Texas Christian University and the writer wishes to express her appreciation to Prof. C. R. Sherer for his help and inspiration and to Prof. Joseph Morgan for his helpful criticisms.

[4] J. H. Jeans, "Electricity and Magnetism" (1927), 54.

NOTES FROM THE NATIONAL BUREAU OF STANDARDS.*

HIGH SPEED ELECTRONIC DIGITAL COMPUTERS.

The National Bureau of Standards has been commissioned by the Bureau of Census and the Office of Naval Research to construct two high speed electronic digital computing machines which may well revolutionize the methods of classification and compilation of data and of numerical computation.

The computers are to be of the automatically sequenced digital type. Automatic sequencing reduces the intervention of a human operator to a minimum while electronic rather than electromechanical or relay type components assure speeds measured in millionths of a second for the arithmetic operations of addition, subtraction, multiplication, and division.

The machines will be capable of handling huge complication problems and of solving the most complicated differential equations in engineering and physics. They will perform, upon instruction, predetermined sequences of calculations running into the hundreds of operations without the intervention of human operators. The result will be the solution of problems in days that would take years if attempted by present standard machines.

In the opinion of outstanding authorities in the field of applied mathematics, wind tunnel and model ship basin testing will be aided by machine computation. At the present time it is only theoretically possible to predict all of the changes involved when an aircraft is in flight, for example. Practically, mathematical solutions to these problems are impossible because it would take years to solve the equations involved by present methods. Answers are now found by laborious tests in wind tunnels and ship basins, observing the action of one part of a craft when certain forces are applied at another part. With the high speed electronic machines much of this information will be available, in a matter of minutes from the computer, in printed form if desired. The complex computations involved are no longer beyond the reach of the applied mathematician.

The machines under construction are to have built-in "intelligence" capable of distinguishing between orders and data presented in coded form. Design specifications call for internal memories in which 5,000 or more 10 or 12 digit numbers and orders combined can be stored. The machines will be able to transfer numbers and orders between their

*Communicated by the Director.

internal organs and memories within a few millionths of a second. Numbers and orders will be taken automatically from a continuous storage medium in the form of a film or tape, and the machines will know when to depend on their internal memories and when to obtain information from this external tape. At predetermined steps in computation, the computers will be able to choose and execute alternative orders upon the basis of the result of the preceding steps. They will know when computations have been completed to a pre-assigned accuracy and will either print the results at that time or store them on an external tape for printing later.

At the present time, a typical Census problem, involving the multiplication of 100,000 pairs of 5-digit numbers followed by summing the products, takes approximately 12 working days exclusive of card handling and data punching. The electronic digital machine for the Census Bureau will handle the same sequence in 10 minutes at the most. A problem of great importance to the Navy is the determination of vibration characteristics of ship machinery and aircraft. One such problem, arising in the flutter analysis of aircraft wings, now takes about 5,000 machine hours by the most efficient method available. With the projected Navy computer, the problem can be solved in a very few hours.

During the recent emergency various federal agencies pioneered in the development and use of ultra-high speed computing machinery. The applications of such machinery range from numerical solutions of problems involving nuclear physics, hydrodynamics, electromagnetic wave propagation, the computation of firing tables, the application of statistical sampling theory and the like, to use as component parts in missile flight simulators and gun-fire control apparatus. The need for high computation speeds was extremely urgent, and the digital machines developed to meet it played a significant role in shortening the conflict.

Federal activity in the computing field stemmed principally from the efforts of the War and Navy Departments to acquire high speed machinery constructed specifically for critical wartime computation needs. There were three significant developments in this period: (1) an electromechanical computing device (the IBM Sequence Controlled Calculator) of the Harvard Computation Laboratory, (2) relay types computing machines of the Bell Telephone Laboratory, and (3) an electronic computing machine (ENIAC) of the Moore School of Engineering of the University of Pennsylvania, designed and built under the direction of Dr. J. W. Mauchly and J. P. Eckhart.

CURRENT PROGRAM.

A significant result of the intensive development and use of the high speed computing machines during the war is that designers have been provided with operating data which would have taken years to accumulate under normal conditions. Experience now indicates rather clearly

both the merits and the deficiencies of existing machines. The consensus of expert opinion is that the potentialities of electronic machines surpass those of the electro-mechanical and relay types and that further development should probably be limited to the electronic field. It is also agreed that an unparalleled opportunity for design improvements is at hand and that electronic digital computing machines can be constructed that will far exceed the ENIAC in speed and flexibility.

Electronic digital computing machine development requires the cooperation of two groups: mathematicians, responsible for the mathematical performance requirements and for organizing the mathematical logic; and physicists and engineers, responsible for developing the basic functional organs and for the engineering design integrating these organs into a unit capable of performing the prescribed tasks. The mathematical phases of the program of the National Bureau of Standards are under the direction of Dr. J. H. Curtiss, assistant to the director and in charge of the Bureau's mathematical and statistical work, while the electronic phases are conducted by the electronics section of the Ordnance Development Division under the direction of Harry Diamond, Chief of the division.

The program involving the Census and Navy machines includes contractual relations with industry. For example, the Raytheon Corporation and the Electronic Control Company have active roles, both organizations having experts in this field. Associated with the Raytheon Corporation is Dr. George R. Stibitz, chief designer of the Bell Telephone large-scale relay computing machines. The Electronic Control Company has been established by Dr. J. W. Mauchly and J. P. Eckhart to design and construct such machines.

A special program is also under way at the Bureau for the Army Ordnance Department on the development of basic organs for computing machines. Close relations exist with the Institute for Advanced Study, Princeton, under Dr. John von Neumann, and the Moore School of Engineering, under Dean Harold Pender. Such diversified cooperation among these principal exponents of electronic digital computers will ensure the development of the most highly advanced machines to meet current needs.

The two machines to be built by the National Bureau of Standards will speed up the Bureau of Census tabulations and compilations and expedite the research program of the Navy Department. The new Census machine will introduce gains of a substantial character. Economy will be effected not only in the cost of tabulation of data but also in the time required to compile Census information. In addition, as a result of quicker processing of raw data, the time between collection of data and reporting them will be shortened. The Navy machine will be available to the armed services and to other government agencies as well as private industry.

COMPUTING LABORATORY.

According to present plans the Navy machine will be located in a large general-purpose computing laboratory, which will also be equipped with standard business-type computing machinery and will be operated by the National Bureau of Standards. This laboratory will be staffed with personnel trained in the methods of numerical analysis. It will contain an analytical section which will conduct basic research in applied mathematics and will assist in formulating problems mathematically and preparing them for numerical solution.

The idea of developing such a laboratory was originally suggested by the Office of Naval Research as a means of expediting research for defense. This type of computing laboratory is a natural outgrowth of the present highly successful Mathematical Table Project of the National Bureau of Standards, consisting of a large computing group located in New York City, and currently supported with funds transferred by the Office of Naval Research. It was established about ten years ago to satisfy the need for more complete and more accurate tables of mathematical functions. Over 28,000 copies of the various tables of the Project are now in use, and the annual demand runs at the rate of about 3,500 volumes.

The project emerged from the war, a fully rounded and versatile computing laboratory with substantial experience in the field of numerical research as well as in the construction of tables. After its incorporation in the larger laboratory mentioned above, two automatic high speed digital calculators will be added to its present complement of electric desk calculators, electric adding machines, automatic electric accounting machines, and IBM punch card machines. The automatic calculators will probably shift the emphasis of the work more strongly than in the past to the analytic aspects because automatic machines will require a vast amount of additional mathematical theory of numerical analysis. The Bureau, in cooperation with the Office of Naval Research, is planning a strong program of research oriented toward such problems, and this program is expected to make a permanent contribution to the scientific resources of the nation.

MEASUREMENT OF THE SLIPPERINESS OF WALKWAY SURFACES.

Slippery walkway surfaces are responsible for a large number of serious injuries and accidental deaths every year. In order to eliminate as far as possible such hazards, a joint research project has been undertaken by the National Safety Council and the National Bureau of Standards directed toward developing safety codes for walkways and footwear. The Council is engaged in collecting data regarding accidents and in making a statistical study to show the circumstances under which ac-

cidents from falls occur. The object of the Bureau's investigation is to develop methods for measuring the slipperiness of walkway surfaces under conditions of use and to apply these methods in selecting materials and methods of maintenance for both walkways and footwear that will reduce the frequency of accidents from falls.

Probably the most common way of improving the antislip characteristics of paving and flooring materials is to provide a roughened surface. This is accomplished in some flooring materials, such as metal stair treads and rubber matting, by forming corrugations or embossments in the surfaces; in others, such as rubber tile, ceramic tile, and troweled monolithic floors, by embedding rough abrasives like alundum or corborundum in the surface. However, due to difficulties of cleaning and maintenance, rough surfaces are shunned for general indoor use.

The establishment of safety codes for walkway surfaces and footwear has been materially handicapped by the lack of adequate methods of measuring slipperiness. Slipperiness is not a constant of the walkway or the footwear but is a function of both surfaces and its study is materially handicapped by the lack of adequate methods of measuring slipperiness. The correlation between coefficient of friction as commonly measured and slipperiness as actually experienced is not good, especially where wet surfaces are involved. Most of the devices that have been used in the past for measurement of the friction between shoe soles and walkway surfaces were concerned with starting rather than sliding friction, and an appreciable area of contact between sole and floor surface was generally used. These methods were thus more representative of a person standing on a floor than of someone in the process of walking, when most slips and falls occur.

As an aid in the design of testing instruments, the National Bureau of Standards is making a study of walking habits in an effort to obtain data on such factors as the manner in which the heel contacts a walkway surface during the restraining phase of a step, the probable angles of contact between heels and walkway surfaces, the momentary forces involved, and the functions of muscles in locomotion. Concealed cameras have been used to take solwmotion pictures of people walking in their usual manner, while they were unaware of being photographed. These pictures show, among other things, that the foot is first placed upon the walkway surface at an angle so that only the rear edge of the heel is in contact during the early stages of the restraining phase of a step. The other foot remains in contact with the walkway and thus bears part of the vertical load until the heel rocks forward and the foot is firmly planted. This phase of the investigation was supplemented by a study of worn heels, which showed that maximum wear usually occurs at the outside border of the rear portion of a heel. The contour of this worn portion was found to be in the form of a curve rather than a bevel.

It is exteemely difficult to reproduce artificially, in the laboratory,

walkway surfaces that are typical of the various conditions in service, because the location of a floor determines to a large extent the texture of its worn surface. Accordingly, P. A. Sigler of the National Bureau of Standards has designed a portable impact-type slipperiness tester that can be used to test floors in actual service. The design is based on the premises that slipping is a function of two surfaces and that in the process of ordinary walking slipping is most likely to occur when the walkway surface is first contacted by the rear edge of the heel.

The instrument is primarily a compound pendulum which sweeps a shoe material over the walkway surface to be tested. A mechanical shoe forms the lower end of the pendulum and is so arranged that a 1½-inch square test piece of rubber, leather, or other heel material can be attached to the underside of the shoe at various angles so that only the rear edge of the test piece makes contact with the walkway surface. A helical spring is used to press the edge of the test piece against the walkway surface during contact. The pendulum is released at a fixed height and the edge of the heel is permitted to sweep over the surface of the floor specimen. A pointer attached to the framework at the center of oscillation of the pendulum indicates on a scale the maximum height to which the pendulum rises above its lowest position. From this information an "antislip coefficient" for the walkway and footwear surfaces can be computed.

Until the many factors which have a bearing on the results can be analyzed, it is advisable to consider the antislip coefficients obtained by this method as relative rather than absolute values. Some of the typical results obtained, however, may be of interest.

The antislip coefficients for concrete blocks ground with carborundum prior to testing ranged from 0.37 to 0.65, whereas the coefficients for a concrete floor with a cement-mortar topping, worn smooth over 25 years of service, ranged from 0.19 to 0.52. These results demonstrate the importance of surface condition and the inadvisability of assigning a single coefficient or even a single range of coefficients to one type of flooring.

The most important differences were observed in the wet state. Rubber tile with a very smooth molded surface showed high coefficients when dry but very low when wet. A rubber tile containing alundum grit was quite similar under dry conditions but was less slippery when wet. A rubber matting containing cotton fibers and in service in a transit bus showed fair antislip characteristics under all of the test conditions.

An investigation was made at Walter Reed General Hospital, Washington, D. C., to determine the relative slipperiness of different treatments for floors on which amputees are learning to walk with crutches and artificial legs. The antislip coefficients obtained for these floors were satisfactory under dry conditions, especially in conjunction

with a rubber heel, but under wet conditions the floors would be classed as extremely hazardous, especially when waxed.

An extensive investigation was also made on the asphalt-tile corridors of the Pentagon Building. The relative order of the antislip coefficients obtained with the portable slipperiness tester correlates very well with their relative slipperiness as actually experienced, indicating that there exists a critical degree of slipperiness which largely determines whether or not a slippery condition exists. With the portable instrument, the critical antislip coefficient is about 0.4. Surfaces having coefficients of 0.45 or better were regarded as satisfactory. Higher antislip coefficients were obtained for waxed asphalt tiles than for untreated tiles when tested with a rubber heel under dry conditions. With a leather heel the opposite was found true except for one type of wax. Under wet conditions all of the corridors would be classed as potentially hazardous for both rubber and leather footwear and especially so when waxed. In general, the antislip properties of the waxed asphalt tile corridors improved with continued exposure to dry maintenance and normal traffic.

In conclusion, it may be stated that good antislip properties under wet conditions are usually associated with asperities which project through the film of water and thus prevent its action as a lubricant. The asperities may represent either the roughened surfaces of the walkway or footwear materials themselves or particles of embedded sand or grit.

SPECTROGRAPHIC ANALYSIS OF PORTLAND CEMENT.

With the advance of knowledge concerning the effects of the several constituents of portland cement upon the properties of concrete, the need has been increasingly felt for more rapid and precise methods for determination of these constituents. Of particular significance in recent years have been the possible effects of the alkali elements. This problem is now under critical observation by various laboratories in connection with studies on the durability of concrete. An investigation by the research fellowship of the Portland Cement Association cooperating with the Spectroscopy Laboratory of the National Bureau of Standards has led to the development of a satisfactory spectrographic method for determination of the constituent elements of portland cement—particularly the minor elements that may contribute favorable or unfavorable properties to concrete.

The usual chemical methods of analysis of portland cement for minor elements are time-consuming and subject to considerable human error, particularly in the determination of the alkali elements. The system of spectrographic analysis, which grew out of earlier investigations applied specifically to the determination of sodium, potassium, and lithium,

provided the solution to this analytical problem. In order to take the fullest advantage of this method, the study was extended by A. W. Helz and B. F. Scribner to include the determination of all the constituent elements of portland cement susceptible of ready determination by spectrographic means.

The principal problem involved in spectrographic analysis is to excite the characteristic spectra of the elements in the sample in a reproducible manner. This was accomplished in the analysis of portland cement by the new method of forming a compressed pellet of the cement mixed with a binder of graphite powder, a reference material of cobalt oxide, and a buffer of potassium nitrate. The pellet and a graphite rod serve as electrodes between which an intermittent arc discharge is passed. The light from the arc is then observed with a spectrograph and the photographed spectrum is measured for the characteristic lines of the elements to be determined. Quantitative determinations are based on comparison with standard samples of cements having compositions carefully determined by chemical methods.

In the sample mixture, the graphite serves a dual purpose: to bind the material together when pressed into the required pellet $\frac{1}{2}$ inch in diameter and $\frac{1}{8}$ inch thick and to conduct electricity through the electrodes. The cobalt oxide provides a reference element in fixed concentration in the sample; the intensities of spectral lines of the minor elements in the cement are measured relative to the intensities of selected cobalt lines. The addition of a known excess of potassium nitrate overcomes effects on the spectra caused by variations in the concentrations of the alkali elements in the cement.

The method provides for the quantitative determination of the following constituents, which are reported as concentration of their oxides: aluminum, iron, magnesium, sodium, potassium, manganese, and titanium. In addition, suitable spectral lines are listed for the detection of lithium, chromium, strontium, and zirconium.

The spectrographic analysis of portland cement provides advantages of speed and freedom from human error. An operator can analyze eight cement samples in duplicate for seven elements, a total of 112 determinations, in 8 hours. The time required for the equivalent chemical analysis would be several days. Furthermore, the method is sufficiently accurate for control work and is believed advantageous for applications in which a considerable volume of testing is involved. Where reliable standard samples are available, the method shows promise of ready extension to the analysis of cement raw materials and many ceramic materials.

THE FRANKLIN INSTITUTE.

MINUTES OF THE STATED MEETING OF THE FRANKLIN INSTITUTE WEDNESDAY MARCH 19, 1947.

The Franklin Institute held its stated meeting in the Lecture Hall on Wednesday, March 19th. Mr. Richard T. Nalle, President of The Franklin Institute, presided at the meeting.

The minutes of the Stated Monthly Meeting for January were printed in full in the February issue of the JOURNAL OF THE FRANKLIN INSTITUTE. The President stated that if there were no corrections or additions to the minutes, they would stand approved as printed. There was no contrary motion.

The Secretary, Dr. Henry B. Allen, was then called upon to give his report on the Membership Department. The following members were elected in the month of February:

Active	55
Associate	39
Student	10
Making a total for February of	104

The Secretary then spoke of the Electron Microscope which The Franklin Institute had purchased from the RCA. The Electron Microscope is to be used in the Laboratories for Research and Development but is to be on exhibition with demonstrations for a period of three weeks.

The President reported that Board of Managers of The Franklin Institute had nominated for honorary membership to The Franklin Institute its two Franklin Medallists for 1947, Dr. Enrico Fermi and Sir Robert Robinson. The By-Laws provide that election should be by four-fifths of the votes of the members present at any stated Institute meeting. Upon motion duly seconded Dr. Enrico Fermi and Sir Robert Robinson were unanimously elected.

The President then called upon Dr. W. F. G. Swann, the speaker of the evening, who spoke on the work done by the Bartol Research Foundation for the past year. He outlined the plan of work and the achievements of the foundation with reference to work with the precision Van de Graaff Generator and progress on the development of a ten-million electron-volt electron accelerator, together with the development on Geiger Counter potentialities and techniques—further developments in cathodes for high-power vacuum tubes. Dr. Swann also spoke of the cosmic ray investigations carried out in a B-29 bomber and the cosmic ray work planned for the National Geographic Society's expedition to Brazil this spring and for subsequent B-29 flights and other flights in higher latitudes. Dr. Swann showed lantern slides depicting some of the work carried on at Bartol.

The meeting was adjourned with a rising vote of thanks to the speaker.

COMMITTEE ON SCIENCE AND THE ARTS.

(Abstract of Proceedings of Stated Meeting held Wednesday, March 12, 1947.)

HALL OF THE COMMITTEE,
PHILADELPHIA, MARCH 12, 1947.

MR. WALTER C. WAGNER in the Chair.

The following report was presented for final action:

No. 3178: Levy Medal.
No. 3179:
No. 3180:

This report recommended the award of a Louis Edward Levy Medal to Everett McMullin Barber, of Beacon, New York; Jay Brent Malin, of Beacon, New York; and Joseph John Mikita, of Wilmington, Delaware, "In recognition of their outstanding paper 'The Elimination of Combustion Knock,' appearing in the April 1946 issue of the Journal of The Franklin Institute."

LIBRARY NOTES.

The Committee on Library desires to add to the collections any technical works that members would wish to contribute. Contributions will be gratefully acknowledged and placed in the library. Duplicates received will be transferred to other libraries as gifts of the donor.

Photostat Service. Photostat prints of any material in the collections can be supplied on request. Orders received in the morning are filled the same day. The average cost for a print 9 × 14 inches is thirty-five cents.

The library and reading room are open on Mondays, Tuesdays, Fridays and Saturdays from 9 A.M. until 5 P.M., Wednesdays and Thursdays from 2 P.M. until 10 P.M.

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NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION.

Effect of Specific Nuclease and Phosphoesterase on Desoxyribonucleic Acid Prepared with the Use of Strong Sodium Hydroxide (Method of Levene).—CHARLES A. ZITTLE. Desoxyribonucleic acid prepared under mild conditions is readily hydrolyzed by specific nuclease so that it becomes soluble in HCl. By subsequent action of phosphoesterase from calf intestinal mucosa the nucleic acid was found to be completely hydrolyzed to nucleosides and phosphoric acid (1). Evidence is presented herein that desoxyribonucleic acid prepared by the method of Levene (2), which involves the use of hot 1.25 N NaOH, is not acted on by the nuclease and is only slowly, although extensively, acted on by the phosphoesterase. Evidence is also presented that the resistance of this nucleic acid to the nuclease concerns the adenine radical.

EXPERIMENTAL.

Preparation of the enzymes and substrates and the manometric and other procedures have been described (1).

About 15 per cent. of the desoxyribonucleic acid prepared by the method of Levene was soluble in 0.25 N HCl initially, but its solubility after prolonged action of the nuclease at pH 7.5 increased to only 20 per cent. Nucleic acid prepared under mild conditions (method of Hammarsten) was made similarly resistant to the nuclease by boiling for 30 minutes with 1.25 N NaOH, a step in the preparation of nucleic acid by the method of Levene. This treatment released 0.9 equivalents of NH_3 per tetranucleotide-residue (four P). The phosphoesterase preparation acting on this material and on the nucleic acid by the method of Levene at pH 7.62, released equal amounts of acid whether silver nitrate, which inhibits adenosine deaminase (3), was present or not. The effect of the phosphoesterase with and without silver nitrate is illustrated in Fig. 1. With the nucleic acid prepared by the method of Hammarsten more acid was measured when silver nitrate was present, since then no deamination takes place of the adenosine formed by the action of the phosphoesterase (both diesterase and monesterase), and, consequently, a partial neutralization of the acid by the ammonia does not occur. The amount of acid released from nucleic acid by the method of Levene was identical whether silver nitrate was present or not. This suggests that the adenosine had been altered, perhaps deaminated, by the strong alkali used in preparing the nucleic acid, or, since phosphoesterase alone cannot completely hydrolyze desoxyribonucleic acid (1), that adenosine had not been liberated.

DISCUSSION.

The negligible effect of the highly specific nuclease on the nucleic acid by the method of Levene suggests that the structure of the latter was altered by the NaOH used in its isolation. This explanation is supported by the change observed in the reactivity of nucleic acid pre-

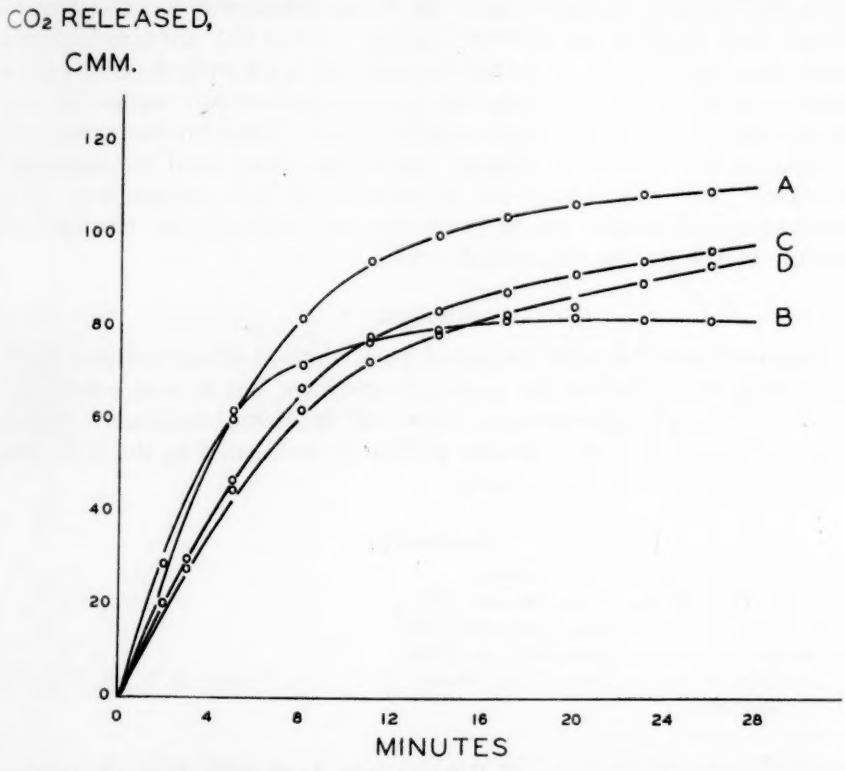

FIG. 1. Effect of phosphoesterase, with and without silver nitrate, on desoxyribonucleic acid prepared with and without the use of strong sodium hydroxide. Experiments were performed manometrically as described (1) with 4.2 mg. of nucleic acid and with 8.0 mg. of phosphoesterase at pH 7.62. Concentration of silver nitrate: 0.00017 moles per liter. Both nucleic acids, by the method of Hammarsten and by the method of Levene, had been previously treated with specific nuclease (1); the solubility of the former in HCl became complete whereas there was negligible change in the solubility of the latter by this treatment. Nucleic acid by the method of Hammarsten: curve A, with silver; curve B, without silver. Nucleic acid by the method of Levene: curve C, with silver; curve D, without silver.

pared by the method of Hammarsten after treatment with NaOH. The negative influence of silver nitrate when the phosphoesterase preparation acts on the nucleic acid by the method of Levene and on the NaOH-treated nucleic acid, and the actual liberation of NH_3 by the alkali treatment, suggest that deamination of the molecule may be

the cause of the alteration in properties. Another possible cause may be that the acid group liberated by the nuclease (1) concerns the adenine radical and that some other alteration in the nucleic acid molecule prevents the initial hydrolysis by the nuclease necessary for the eventual liberation of adenosine by the phosphoesterase.

The inability of the nuclease to act on the alkali-treated (deaminated?) nucleic acid suggests that the bond specifically broken by this enzyme may involve the adenine group. Since the adenine group is also in ribonucleic acid, all of the bonds of which are hydrolyzed by phosphoesterase (4), a different bonding is suggested for the nucleotide containing this group in desoxyribonucleic acid. Desoxyribonucleic acid deaminated by the use of nitrite, which has been used to deaminate ribonucleic acid (5), would be of interest in this connection. This deamination procedure would probably not degrade the nucleic acid molecule to the degree that alkali does.

SUMMARY.

Desoxyribonucleic acid prepared by a method which utilizes strong NaOH was not acted on by a specific nuclease but it was extensively hydrolyzed by phosphoesterase from calf intestinal mucosa. Experiments indicated that the adenine radical is concerned in the resistance of the nucleic acid to the nuclease.

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Abstract of **Hydrolysis of Ribonucleic Acid with Phosphoesterase from Calf Intestinal Mucosa.**—CHARLES A. ZITTLE (*Journal of Biological Chemistry*, **166**:491, 1946). Ribonucleic acid was completely hydrolyzed, with the liberation of all of the phosphoric acid, by an enzyme preparation from calf intestinal mucosa. The number of acid groups liberated during the hydrolysis was estimated by manometric and titrimetric procedures. It was found that the number of secondary phosphoric acid groups liberated from the most highly polymerized preparations of nucleic acid approached four per tetranucleotide residue.

Abstract of **Adenosine Deaminase from Calf Intestinal Mucosa.**—CHARLES A. ZITTLE (*Journal of Biological Chemistry*, **166**:499, 1946). An enzyme preparation from calf intestinal mucosa is described which deaminates adenosine and desoxyriboadenosine. The activity of the

deaminase is conveniently measured manometrically; the use of the enzyme for estimating adenosine is suggested. The enzyme preparation contains phosphoesterases but these were inactive at pH 5.9, whereas the deaminase has considerable activity. The activity of the deaminase with adenosine as the substrate has a broad pH optimum at 7.0; the deaminase is very sensitive to silver, 4.0×10^{-7} moles per liter causing 50 per cent. inhibition.

BOOK REVIEWS.

QUALITY CONTROL CHART TECHNIQUE WHEN MANUFACTURING TO A SPECIFICATION, by B. P. Dudding and W. J. Jennett. 74 pages, charts, 14 × 22 cms. London, The General Electric Co., Ltd., 1945. Price $1.00 (paper).

This pamphlet is devoted to procedures found acceptable when introducing and operating control chart technique as a factory routine. It is a guide for those concerned with machine manufacture of articles to dimensional limits. The first part is on utilizing the actual measurements made on samples selected regularly from the production of a machine or process (quantitative data), and the second part deals with the utilization of the results of classifying a product into catagories; for example, "defective" and "effective" (qualitative data). Both parts contain directions which should be followed when first attempting to introduce control charts and directions for constructing control charts when manufacturing to specified limits.

R. H. OPPERMANN.

MICROCALORIMETRY by W. Swietoslawski. 199 pages, illustrations, 16 × 24 cms., New York, Reinhold Publishing Corporation, 1946. Price $4.75.

Recent progress in physics, physical chemistry, physiology and biology as well as in some branches of industry has stimulated the development of a special branch of calorimetry, which deals with measuring the very small thermal effects produced by different objects. The variety of objects is great and they include the evolution of heat by adsorption or absorption of gases and vapors, by the slow recrystallization of metallic alloys, and by the evaporation of small amounts of volatile liquids. The heat produced by a growing plant, by bacteria, and by germination of seeds are other examples. This book describes the methods used in measuring these small amounts of heat.

The work begins by pointing out the importance of recognizing the duration of the thermal process as a directive in selecting the type of calorimeter to be used. The part taken by calorimeter jackets, thermostats and thermometers in microcalorimetric measurements is explained with an explanation of apparatus and its theory of operation. The history of the well known method of comparative measurements is treated, leading to its use in microcalorimetry with the result of avoiding errors by reducing to a minimum the number of corrections for secondary phenomena. The use of twin calorimeter; the carrying out of two successive measurements in one calorimeter; the development of heat by the object only and then by the object and an electric current; and comparing heat liberated with that produced by a standard are methods described in this book.

Chapter headings in the rest of the book are indicative of the contents—Twin Microcalorimeters Used in Radiology involving methods which form a natural transition from the microcalorimetric methods used in the past to the new technique based on using isothermal or adiabatic jackets; Use of the Ice Calorimeter in Microthermal Measurements concluding that this apparatus is still far from being a convenient one for this work; Tian's Investigations in Microcalorimetry; Microthermal Measurements of Processes of Short Duration; Adiabatic Microcalorimeter; Static and Kinetic Methods of Microthermal Measurements; Labyrinth Flow Calorimeter, an instrument offering many advantages. The last chapter gives a comprehensive description of the methods which have been used up to this time for the purpose of facilitating the proper choice of a method to be employed in any particular case.

The book contains a list of the many references discussed, and author, and a subject index. It should assist greatly in the development of microcalorimetric methods for the growing number of requirements.

R. H. OPPERMANN.

PETROLEUM PRODUCTION, by Park J. Jones. Volume II. 293 pages, diagrams, 16 × 24 cms. New York, Reinhold Publishing Corporation, 1946. Price $4.50.

The end sought for in petroleum production depends on economics and the economic limits for wells and reservoirs depend on producing and operation methods which include number and location of producing and injection wells, completion of wells, and per-well rates of production and injection. All wells reach economic limits before they are abandoned and the interest of the operator is primarily the prevention of premature economic limits. The book at hand discusses theories and characteristics of oil wells.

The maximum rate of production is the first topic and the treatment thereafter deals with development periods, economic factors, rates of depletion and production, and the optima for uniform and variable rates of depletion. The presentation is direct and plain. It should appeal to those interested in petroleum production.

R. H. OPPERMANN.

BULLETS BY THE BILLION, by Wesley W. Stout. 77 pages, illustrations, 13 × 21 cms. Detroit, Chrysler Corporation, 1946.

This booklet is a testimonial to American industrial genius. It is a story of how the Chrysler Corporation, Evansville, Indiana plant, was converted from automobile assembly to the manufacture of small arm ammuniton and the contribution it made to the war effort. This plant made more than three billion .45 caliber cartridges; almost a half a billion .30 caliber cartridges; hundreds of thousands of rounds of other special types of ammunition; specially packed a billion and a half rounds of ammuniton for use in the Pacific; reconditioned 1,662 General Sherman tanks; rebuilt 4,000 army trucks; delivered 800,000 tank grousers and when the war ended, had just got under way on an order for 7,000,000 firebombs. The book makes inspiring reading and gives an insight to the power of industrial America.

R. H. OPPERMANN.

TEXTILE CHEMICAL SPECIALTY GUIDE 1946–47 edition, 408 pages, 15 × 23 cms. New York, Textile Book Publishers, Inc., 1946. Price $5.00 in U. S. A. and Canada; $6.00 foreign countries (paper).

This well-known buyers' guide to textile chemical products is now appearing in the fourth edition and the contents are arranged in the following manner. The names of the manufacturers and their textile chemical products are listed first, followed by a classified list of these products according to their use and applications. Finally an alphabetical list of brand names and registered trade names. This field of chemistry has become very extensive in recent years and the guide presents a quick and compact method of finding information that normally would require an extensive search in trade literature. No matter what little information a person may have of a chemical in this field there is no doubt after consulting this guide that he will be able to secure complete information from the three well arranged lists.

This book is a valuable guide for the buyer and user to find both the products needed and its manufacturer.

W. P.

FIRE AND THE AIR WAR, edited by Horatio Bond. 262 pages, drawings and illustrations, 16 × 23 cms. Boston, National Fire Protection Association, 1946. Price $4.00.

The knowledge and experience gained in World War II relating to production and destruction are being reexamined and changed to peacetime uses. Among the most important subjects are the lessons forthcoming from the part that fire played in the war. It is no news that great destruction was caused in the cities of the world by fire and by high explosives. Fire protection people have a great stake in understanding what happened. The destruction was seldom less than two-thirds fire damage and in many cities, notably those of Japan the damage was practically one hundred per cent fire damage.

This book contains papers on fire in the war, its effectiveness in destruction, means of combatting it, and the lessons on precaution derived from the experience. Written by author-

ities on the subject, they make reference to England including town planing lessons of the fire blitz, the wartime fire service, and reorganization of the British fire service; to Germany citing incendiary warfare, fire attacks, German fire departments under air attack, and fire casualties of the German attacks; to Japan in the action of Japanese fire departments under air attack, and lessons of the Japanese attacks. The Official Report of the U. S. Strategic Bombing Survey on the effects of atomic bombings of Hiroshima and Nagasaki is given. A summation of the knowledge gained and the start toward its use in peacetime is given in the last paper.

This series of papers warrants attention of the layman, city planners, architects, fire protection engineers, and fire fighters.

R. H. OPPERMANN.

COMMUNICATION THROUGH THE AGES, by Alfred Still. 201 pages, illustrations, 14 × 21 cms. New York, Murray Hill Book, Inc., 1946. Price $2.75.

The historical account of the science and art of communication is prefaced by preliminary speculations as to what it is that can be transmitted over ever increasing distances with ever increasing speed. Analysis reveals that language is not a perfect means of communication, as there are few words in any language that have not several meanings or shades of meaning. Actually, language is a sort of code. Being no more than signs or symbols, words do not invariably convey the same meaning to all men. Language is a convenient medium for conveying information but it also serves admirably for concealing the truth. This, in the very beginning, serves to introduce the reader to an interesting and thought provoking presentation.

That which follows relates the history from when drums, fire, and smoke were used, through the use of rays of light into the era of electricity, the telephone, and radio. This is not just a mere statement of facts in proper order. The style used is one of a person speaking, drawing on many works of others in a logical order rather than chronological. At the end of the book, as in the beginning, the reader is taken into the speculative realm, this time on a discussion of communication without words. In telepathy, the knowledge gained is immediate knowledge. It is not dependent upon language or the correct interpretation of words. It is a means of communication which was probably better known if not better understood by unsophisticated primitive man than by the modern psychologist. The author asks, "Would an extended and more thorough study of telepathy be helpful?" The answer is, "That depends not only upon the method of investigation but also its aim . . . if the investigator will diligently seek the underlying principles which govern natural phenomena as telepathy or clairvoyance, he may chance upon new avenues to much valuable knowledge . . ." Here is a true boundary of knowledge beyond which is a vast field for pioneering.

The back of the book contains a chronological table of events dating from 1084 B.C., a name index and subject index. There is much in this book for informative recreation.

R. H. OPPERMANN.

PUBLICATIONS RECEIVED.

Servomechanism Fundamentals, by Henri Lauer, Robert Lesnick and Leslie E. Matson. First Edition. 277 pages, drawings, 16 × 23 cms. New York and London, McGraw-Hill Book Company, Inc., 1947. Price $3.50.

First-Year Qualitative Analysis, by Carl J. Likes and Aubrey E. Harvey, Jr. 134 pages, drawings, 14 × 22 cms. New York, Thomas Y. Crowell Company, 1947. Price $1.25 (paper).

Actions of Radiations on Living Cells, by D. E. Lea. 402 pages, drawings, tables and illustrations, 15 × 22 cms. New York, The Macmillan Company, 1947. Price $4.50.

The Chemistry of Commercial Plastics, by Reginald L. Wakeman. 836 pages, tables, and illustrations, 15 × 24 cms. New York, Reinhold Publishing Corporation, 1947. Price $10.00.

Physical Constants of Hydrocarbons, by Gustav Egloff. Volume IV. American Chemical Society Monograph Series. New York, Reinhold Publishing Corporation, 1947. Price $17.50.

CURRENT TOPICS.

Drydock Launching. (*The Welding Journal*, Vol. 24, No. 10.)—While thousands of Pittsburghers looked on Saturday, August 10th, the largest vessel ever to be built on the inland rivers of America was launched into the Ohio River. The vessel is the ARD-33, a floating drydock designed by the Bureau of Yards and Docks, Navy Dept., for the use of the Bureau of Ships. It was built under the supervision of Bureau of Yards and Docks by Dravo Corp. at its Neville Island Yards at Pittsburgh, Pa., for the U. S. Navy. This ship-lifting giant is 448 ft. long by 97 ft. wide and 45 ft. high. When it is completely outfitted it will leave the Dravo yards under special tow and be taken 2,000 miles by water through the locks of the Ohio River and down the Mississippi. Defined by the Navy as an auxiliary repair dock, ARD-33 will have a lifting capacity of 6,000 tons. It is particularly designed to serve auxiliary or tender type ships as well as Liberty Ships. These vessels in many cases were formerly commercial type ships built for or put to Navy use during World War II.

After the launching of ARD-33, the hull was towed to an outfitting dock where work of completing the vessel is now under way. The structure is butt welded and contains more than 73 miles of welding. It is non-self-propelled, since drydocks of this class are moved infrequently. The vessel is designed to accommodate a crew of 130 men and 7 officers.

To test mechanically the operation of drydock before it is sent to sea, dredging has been done in the Ohio River near Rochester, Pa., to provide a depth sufficient to enable submergence tests to be made. While these river tests will accurately determine the time required to submerge and emerge, it is believed that the dock can be lowered to receive a vessel in less than 30 min. and subsequently emerge by pumping out its tanks in about 2 hr.

The drydock is a self-contained unit. It has its own water distillation plant, sanitary disposal facilities, diesel electric generator sets for light and power, and the crew quarters are such that the men will be comfortable in any climate. Machinery spaces, workshops and crew quarters are contained in the 12-ft. wide sidewalls. Crew quarters are modern and recreational facilities include a soda fountain and moving picture project. The tremendous storage capacity available for food, repair parts and material means the vessel could operate many months at remote locations with limited tender or supply service. A watertight longitudinal bulkhead runs the entire length of the hull. Eight watertight tanks on each side provide the means for submerging and raising the dock. Each of the 16 tanks is served by a pump and flooding valve and all tanks can be filled or pumped out simultaneously, if need be, as control is maintained from a central station. An indicating system is provided that shows the dockmaster the depth of water in all compartments at all times.

R. H. OPPERMANN.

Nickel Coinage.—In discussing the nickle industry for the year 1946, Mr. Robert C. Stanley, Chairman and President of The International Nickel Company of Canada, Limited, stated,

"Among the outstanding developments in nickel coinage during 1946 were the substitution of pure nickel for silver coinage in India; the replacement of silver coinage by a cupro-nickel issue in the United Kingdom; the resumption by the United States of minting the pre-war five-cent piece containing one-quarter nickel and the resumption by Canada of the issuance of pure nickel five-cent pieces.

"Pure nickel is well suited for coinage because of its appearance, wearability, the difficulty counterfeiters find in working it, and the fact that it is magnetic and readily distinguishable from most other coins. The metal has been used for coinage purposes since 1850. Of the 139 coin-issuing governments for which statistics are available, over one hundred have used nickel either in its pure state or as an alloy, as a part of their coinage systems at some time during their monetary history.

"The British coinage bill authorized the issuance of cupro-nickel coins in the same denominations as the country's existing silver coinage. The cupro-nickel coins will be composed of 75 per cent. copper and 25 per cent. nickel. The present quaternary alloy silver coins consist of 50 per cent. silver, 40 per cent. copper, 5 per cent. nickel and 5 per cent. zinc. The advantages of using pure nickel instead of cupro-nickel in place of the silver coinage were fully discussed during the Parliamentary debates on this subject in October. It developed that majority opinion was willing to accept cupro-nickel rather than pure nickel as a temporary expedient because the machinery and equipment in the British mint would require various modifications to handle pure nickel but was suitable at present for stamping out cupro-nickel coins.

"Many other countries are contemplating the use of, or return to, pure nickel or nickel alloy coinage."

R. H. O.

Radar Techniques Useful in Industrial Field.—Although industrial electronic engineers and communication engineers have long felt that their circuits and techniques had little in common, the development of such special communication fields as radar and loran has brought about many new circuits which are useful in the industrial field. So said General Electric Engineer W. D. Cockrell in a paper entitled "Radar Techniques In An Industrial Control" presented before the AIEE in New York.

Square-wave circuits, clippers, blocking, timing, and counting circuits, normally associated with radar, have an important place in industrial electronic circuits, Mr. Cockrell continued. To illustrate this he described a photoelectric side-register control, used so widely in the paper and printing industry and employing many "radar-like" circuits, which was designed before the war.

In this application, a photoelectric scanning head views the strip edge or printed line. Any deviation from the correct path is signalled to the electronic control pannel which excites an amplidyne generator. The amplidyne drives a motor which moves the feed roll sidewise to correct the position or "register" of the strip.

By describing in detail the various electronic circuits in this industrial application, Mr. Cockrell showed that the elementary electronic tube circuits familiar to the radar man can be combined and arranged to serve many func-

tional requirements. He concluded by saying that in industrial electronics the application engineer gets the most satisfactory results by breaking a problem down to its basic functions so that each tube circuit has but one job to do. These basic elements may then be combined to obtain the final results.

R. H. O.

Wartime Knowledge is being Harnessed for Peacetime Production.—Dr. L. W. Chubb, Director of the Westinghouse Research Laboratories recently declared, "The year 1946 was especially notable in one important respect—the large-scale cooperative investigation into nuclear physics with the eventual objective of converting atomic energy into useful, rather than destructive, work. Easily the most spectacular outgrowth of World War II, the success of the atomic bomb led large numbers of people to believe that the power of the atom was just about ready for equally spectacular peacetime work.

"But informed scientists and others who worked closely with the atomic projects knew there were many problems to be overcome before this unparalleled source of power could take its place as a useful, working member of society. The search for these answers has developed into the greatest cooperative group-research program in history with private industry, the government and educational institutions all contributing to its effectiveness.

"The activities of only one concern serve to illustrate the extent of this endeavor. During 1946 Westinghouse sent research engineers and physicists to join the pool of scientific manpower working on peacetime applications of atomic energy. One of these men, Dr. W. A. Johnson, manager of the metallurgical section of our Laboratories, was selected to set up and direct a new metallurgical division of the Clinton Laboratories at Oak Ridge, Tenn. You might call this project a 'super-university of the atomic age.'

"In our own Laboratories in East Pittsburgh, we are continuing the nuclear research that was interrupted by the war and to do this have completely modernized the electrostatic 'atom-smasher.' In past years this apparatus proved invaluable in our fundamental research program. With its aid were made such discoveries as photo-fission, the splitting of uranium atoms by high-energy gamma rays with an accompanying release of large amounts of energy. We are hopeful it will enable us to make even greater contributions to nuclear science in the future."

For the Bikini atom bomb tests last summer the Westinghouse Research Laboratories coordinated much equipment and test procedures, Dr. Chubb said. He commented that "our engineers just about turned the electronics laboratories into a miniature radio factory for the production of much of this equipment."

Dr. Chubb emphasized, however, that while atomic research was one of the foremost projects it was by no means interfering with investigations along other lines. Development work is continuing, he said, on such subjects as improvements in metals and plastics; gas turbines and jet propulsion equipment; more efficient motive power for the nation's railroads and devices to insure the comfort of passengers; better methods of welding; and improved insulation for electrical equipment.

"All in all," Dr. Chubb commented, "I believe we may consider 1946 as

a year of consolidation during which our scientists and engineers were given an opportunity to digest the results of wartime research and plan ways to use this new knowledge for peacetime purposes. As we look back I am certain we will think of this year as one in which we able to lay a solid foundation for the gains of the future."

Although the nation was at peace for the entire year, many of the outstanding scientific and engineering achievements revealed during 1946 were born of wartime necessity. Some were useful only for military purposes but others are already busy at their peacetime tasks. In the latter category come such developments as the all-steel motor, built for the Navy to withstand the shock of heavy guns firing and the tremendous concussion of a bomb blast; and numerous special lighting devices, built to lessen the hazards of military aviation.

R. H. O.

Shot Peening Increases Durability of Metal Parts. (*Compressed Air Magazine*, Vol. 52, No 1).—Shot peening will greatly increase the service life of engine cooling fans, exhaust-valve rocker arms, and many other parts of steel machinery, according to a report prepared for the National Defense Research Committee by R. L. Mattson and J. O. Almen of the Research Laboratories Division, General Motors Corporation. Experiments of which the report is based were conducted over a 2-year period.

Numerous parts, as well as bar-stock test specimens of steel and other metals, were given the shot treatment and then studied. Primary interest was in fatigue durability, but other properties such as static strength, impact resistance, hardness, friction, corrosion resistance, surface roughness, and surface failures also were tested. It was discovered that shot peening lengthened the service life of torsion bar springs by from 700 to 900 percent and trebled the durability of engine cooling fans.

Shot-peened exhaust-valve rocker arms were found to stand up several times as long as polished ones, and the peening of undersized rocker arms made them equal in this respect to unpeened polished rocker arms of standard size. Peening also imparted greater service life to some steel gears, as well as to specimens of aluminum-alloy sand castings, cold-drawn nickel and monel, and soft inconel. It was determined that shot peening permits manufacturing springs from steel harder than that normally used for the purpose. Several types of machine parts gave inconclusive or negative results. The durability of a tractor final-drive pinion was reduced by about 60 per cent., as peening caused pitting that induced failure.

Shot peening consists in cold working metal by pelting the surface with shot by means of equipment that is basically the same as that used in shot blasting metal to clean it. The shot acts like myriads of tiny hammers that compact the surface and bring about beneficial structural changes.

The Mattson-Almen 134-page report is sold by the Office of Technical Services, Department of Commerce, Washington 25, D. C., in either photostat or microfilm form at $9 and $3, respectively.

R. H. O.